1 Stellar Evolution

How is it that we can know so much about stars despite the fact that we cannot travel to them?

1.1 Stellar Models

Stars are simple objects.

Using the laws of physics and computers, we learn much.

Four Laws of Stellar Structure

1. Hydrostatic Equilibrium - The weight of each layer is balanced by the pressure in that layer.

2. Energy Transport - Energy moves from hot to cool by radiation, convection or conduction.

3. Continuity of Mass - Total mass equal the sum of the shell masses, no gaps allowed.

4. Continuity of Energy - Total luminosity equals the sum of the energies generated in each shell.

The Equations 4 coupled differential equations

\[
\begin{align*}
\frac{dM}{dr} &= 4\pi r^2 \rho \\
\frac{dP}{dr} &= -\frac{GM}{r^2} \rho \\
\frac{dL}{dr} &= 4\pi r^2 \rho e \\
\frac{dT}{dr} &= -3 \frac{\kappa \rho L}{16\pi ac T^3 r^2}
\end{align*}
\]
1.1.1 Understanding the Main Sequence

All stars are not the same.

We try to understand high mass stars, average stars, and low mass stars using stellar models.

One of the most fundamental relationships that is known about main sequence stars is

**The Mass - Luminosity Relation**  This relates the rate of thermonuclear fusion to the star’s mass.

The luminosity of the star (in solar luminosities) is due to the rate at which hydrogen is being fused into helium and in the process converting mass into energy.

\[ L = M^{3.5} \]

High mass stars emit tremendous amounts of energy. Low mass stars emit small amounts of energy.

High mass stars convert more mass into energy every second than a low mass star. Even though they have more mass, they use up their fuel in a shorter amount of time.

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### Table of Values

<table>
<thead>
<tr>
<th>( \frac{R}{R_\odot} )</th>
<th>( T ) (10^6K)</th>
<th>Density (g/cm³)</th>
<th>( \frac{M}{M_\odot} )</th>
<th>( \frac{L}{L_\odot} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.006</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.90</td>
<td>0.60</td>
<td>0.009</td>
<td>0.999</td>
<td>1.00</td>
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<tr>
<td>0.80</td>
<td>1.2</td>
<td>0.035</td>
<td>0.996</td>
<td>1.00</td>
</tr>
<tr>
<td>0.70</td>
<td>2.3</td>
<td>0.12</td>
<td>0.990</td>
<td>1.00</td>
</tr>
<tr>
<td>0.60</td>
<td>3.1</td>
<td>0.40</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>0.50</td>
<td>4.9</td>
<td>1.3</td>
<td>0.92</td>
<td>1.00</td>
</tr>
<tr>
<td>0.40</td>
<td>5.1</td>
<td>4.1</td>
<td>0.82</td>
<td>1.00</td>
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<tr>
<td>0.30</td>
<td>6.9</td>
<td>13.</td>
<td>0.63</td>
<td>0.99</td>
</tr>
<tr>
<td>0.20</td>
<td>9.3</td>
<td>36.</td>
<td>0.34</td>
<td>0.91</td>
</tr>
<tr>
<td>0.10</td>
<td>13.1</td>
<td>89</td>
<td>0.073</td>
<td>0.40</td>
</tr>
<tr>
<td>0.00</td>
<td>15.7</td>
<td>150</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Zero Age Main Sequence - ZAMS

Main Sequence Stars

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>Mass (Sun = 1)</th>
<th>Luminosity (Sun = 1)</th>
<th>Approximate Years on Main Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>O5</td>
<td>40</td>
<td>405,000</td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>B0</td>
<td>15</td>
<td>13,000</td>
<td>$11 \times 10^6$</td>
</tr>
<tr>
<td>A0</td>
<td>3.5</td>
<td>80</td>
<td>$440 \times 10^6$</td>
</tr>
<tr>
<td>F0</td>
<td>1.7</td>
<td>6.4</td>
<td>$3 \times 10^7$</td>
</tr>
<tr>
<td>G0</td>
<td>1.1</td>
<td>1.4</td>
<td>$8 \times 10^7$</td>
</tr>
<tr>
<td>K0</td>
<td>0.8</td>
<td>0.46</td>
<td>$17 \times 10^7$</td>
</tr>
<tr>
<td>M0</td>
<td>0.5</td>
<td>0.08</td>
<td>$56 \times 10^9$</td>
</tr>
</tbody>
</table>

**Life Expectancy**  The time that a star is on the main sequence depends on two things:

1. The amount of hydrogen available for converting to helium
2. The rate at which hydrogen is being converted to helium

If \( T \) is the life expectancy of a star in solar life-times then

\[
T = \frac{M}{L} = \frac{M}{M^{3.5}} = \frac{1}{M^{2.5}} = M^{-2.5}
\]

Work an example.

1.2 Post Main Sequence

Mass determines the fate of a star.

Stars are like campfires, ashes accumulate at the center if not mixed.

The accumulating helium in the core contracts due to its own mass, causing the core to grow hotter, but not hot enough to fuse helium into carbon. Temperatures are hot enough for a hydrogen shell surrounding the core to continue fusion into more helium. The star is currently overproducing energy, hydrogen in the shell and the conversion of gravitational energy into thermal energy. This overproduction causes the star to expand its outer layers, thereby, cooling them and the star becomes a giant.
How much can the helium core contract?

**Degenerate Matter**  
Electron degeneracy - Pauli Exclusion Principle  
Fermions cannot be in the same energy at the same time in the same place. Helium core contracts to a minimum size and then stops.
A degenerate gas resists compression
Degenerate gas - does not obey the Pressure - Temperature Themostat (pressure does not depend on temperature).

**Helium Fusion** If the temperature is hot enough, helium fusion can take place.
Triple Alpha Process - fusing 3 helium atoms to form Carbon

\[
\begin{align*}
\frac{4}{2}He + \frac{4}{2}He & \rightarrow \frac{8}{4}Be + \gamma \\
\frac{8}{4}Be + \frac{4}{2}He & \rightarrow \frac{12}{6}C + \gamma
\end{align*}
\]

Helium fusion can take place either fast or gradually.
If gradual, no significant effects, only more energy generation so star expands even more.
Helium Flash - helium explosively fuses helium in the core, does not happen to all stars
Star is not destroyed and explosion is unseen to the outside, outer layers absorb the energy of the explosion
Stars less than 0.4 solar masses are not massive enough to ignite helium fusion in their cores.
Stars more than about 3 solar masses ignite the helium gradually before the core becomes degenerate.
Medium mass stars experience the helium flash.
Medium mass stars can produce planetary nebula.

Medium mass stars end their lives as white dwarfs, wd.
Eventually, all of the helium will have fused into carbon and oxygen but the core can not collapse any further nor the temperature is not high enough to fuse carbon into oxygen and silicone.
Carbon fusion needs stars > 3 solar masses which can generate temperatures of 600,000,000 K
Star Clusters  Allows astronomers to learn about stellar evolution.

Properties of star clusters
The following is a list of the relationships between the stars in a cluster

1. All stars are of the same distance (approx.) - need only plot apparent visual magnitude, $m_v$, vs. temperature

2. All formed from the same nebula

3. All have the same chemical composition

4. All formed at about the same time

Turnoff point  The masses of the stars at the turnoff point gives the age of the cluster.

The life expectancy of the stars at the turnoff point equals the age of the cluster.

<table>
<thead>
<tr>
<th>Nuclear Fuel</th>
<th>Nuclear Products</th>
<th>Minimum Ignition Temperature</th>
<th>Main-Sequence Mass Needed to Ignite Fusion</th>
<th>Duration of Fusion in a 25$M_\odot$ Star</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>He</td>
<td>$4 \times 10^4$ K</td>
<td>0.1 $M_\odot$</td>
<td>7 $\times 10^9$ yr</td>
</tr>
<tr>
<td>He</td>
<td>C, O</td>
<td>$120 \times 10^4$ K</td>
<td>0.4 $M_\odot$</td>
<td>0.5 $\times 10^9$ yr</td>
</tr>
<tr>
<td>C</td>
<td>Ne, Na, Mg, O</td>
<td>$0.6 \times 10^4$ K</td>
<td>4 $M_\odot$</td>
<td>600 yr</td>
</tr>
<tr>
<td>Ne</td>
<td>O, Mg</td>
<td>$1.2 \times 10^4$ K</td>
<td>$-8$ $M_\odot$</td>
<td>1 yr</td>
</tr>
<tr>
<td>O</td>
<td>Si, S, P</td>
<td>$1.5 \times 10^4$ K</td>
<td>$-8$ $M_\odot$</td>
<td>~ 0.5 yr</td>
</tr>
<tr>
<td>Si</td>
<td>Ni to Fe</td>
<td>$2.7 \times 10^4$ K</td>
<td>$-8$ $M_\odot$</td>
<td>~ 1 day</td>
</tr>
</tbody>
</table>
Two types of clusters

1. Open clusters - found mainly in the disk of the galaxy
2. Globular clusters - found in the halo of the galaxy
Variable Stars  When stars reach the giant stage their interiors can be highly unstable.

They enter the instability strip on the $H - R$ diagram.

The star can change its brightness over a period of a few days to a few months.

These stars change their brightness by pulsating - i.e., expanding and contracting over time.

A layer of helium cycles between transparent and opaque over time.

The star acts like a mass on a spring which has been disturbed from its equilibrium position.
Types of Variable Stars
1. Cepheid Variables - named after δ Cephei, Period of 5.37 days
   They are supergiant and bright giant stars of spectral type F or G.
   Periods range from 2 days up to about 60 days.
   Polaris is a Cepheid variable.
   Two types of Cepheids -
   (a) Type I Cepheids - have chemical composition similar to Sun, contains elements heavier than helium
   (b) Type II Cepheids - have chemical composition of only hydrogen and helium.

2. RR Lyrae Variables - Periods of less than a day.
   Fainter than Cepheids

**Variable Stars are important distance indicators**
There exists a Period - Luminosity relation - connects the period of the variation to the luminosity (absolute magnitude, M) of the star.
For a classic Population I type Cepheid variable star

\[ M_v = -2.81 \log_{10}(P) - (1.43 \pm 0.1) \]

where \( P \) is measured in days.

1.2.1 Variable stars can be used to find distance

Luckily, variable stars are supergiant stars so we can see them from great distances. Distances to galaxies can be determined.

Classical Cepheids are yellow supergiants of spectral class F6 – K2 and their radii change by (~25% for the longer-period Cepheid variables) which is millions of kilometers during a pulsation cycle.