EGR 261
Signals and Systems
File: Frequency Response.doc

Frequency Response using Bode plots, MatLab, Excel, and MathCad

There are two approaches to generating frequency response graphs (log magnitude and phase plots):

1) **Exact plots -** Any graphing program such as Excel, MATLAB, or MathCad can be used to generate exact plots for both the log magnitude and the phase.
2) **Bode plots -** Bode plots give "straight-line approximation" to the log magnitude and phase plots. These approximations are reasonably accurate. Bode plots have the added benefit of allowing the engineer to clearly understand where each break points occurs, what effect each break point has, and perhaps how to adjust the break points in order to achieve some desired response.

Example: Consider the transfer function $H(s) = \dfrac{800(s + 100)}{(s + 4000)}$

A) Plot the Bode log magnitude (LM) and Bode phase plots
B) Plot the exact LM and phase plots using Excel
C) Plot the exact LM and phase plots using MATLAB
D) Plot the exact LM and phase plots using MathCad

A) **Bode Plot Solution**:

First form H(jw): $H(jw) = \dfrac{800(jw + 100)}{(jw + 4000)}$

Now put H(jw) into "standard form": $H(jw) = \dfrac{20(1 + j\frac{w}{100})}{(1 + j\frac{w}{4000})}$

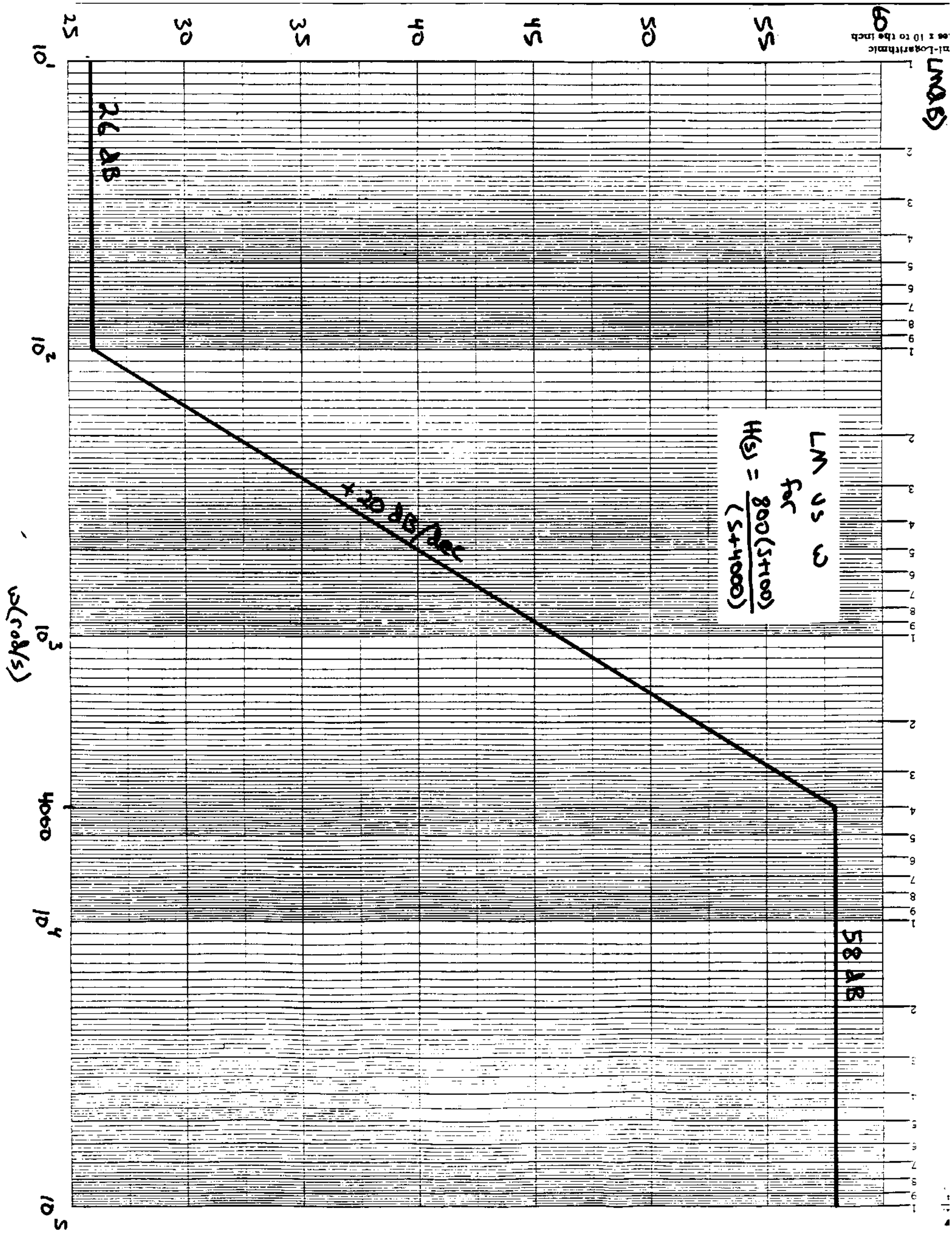
- The first break in H(jw) occurs at w = 100. If the Bode plots are to begin at least one decade before the first break, then the graph should start at w = 10.
- The last break in H(jw) occurs at w = 4,000. If the Bode plots are to end at least one decade after the last break, then the graph should end at w = 40,000 or later. If complete cycles are to be used, the graph should end at w = 100,000.
- A graph from w = 10 to w = 100,000 (or from 10^1 to 10^5) will require 5 - 1 = 4 cycles.

Bode LM plot:

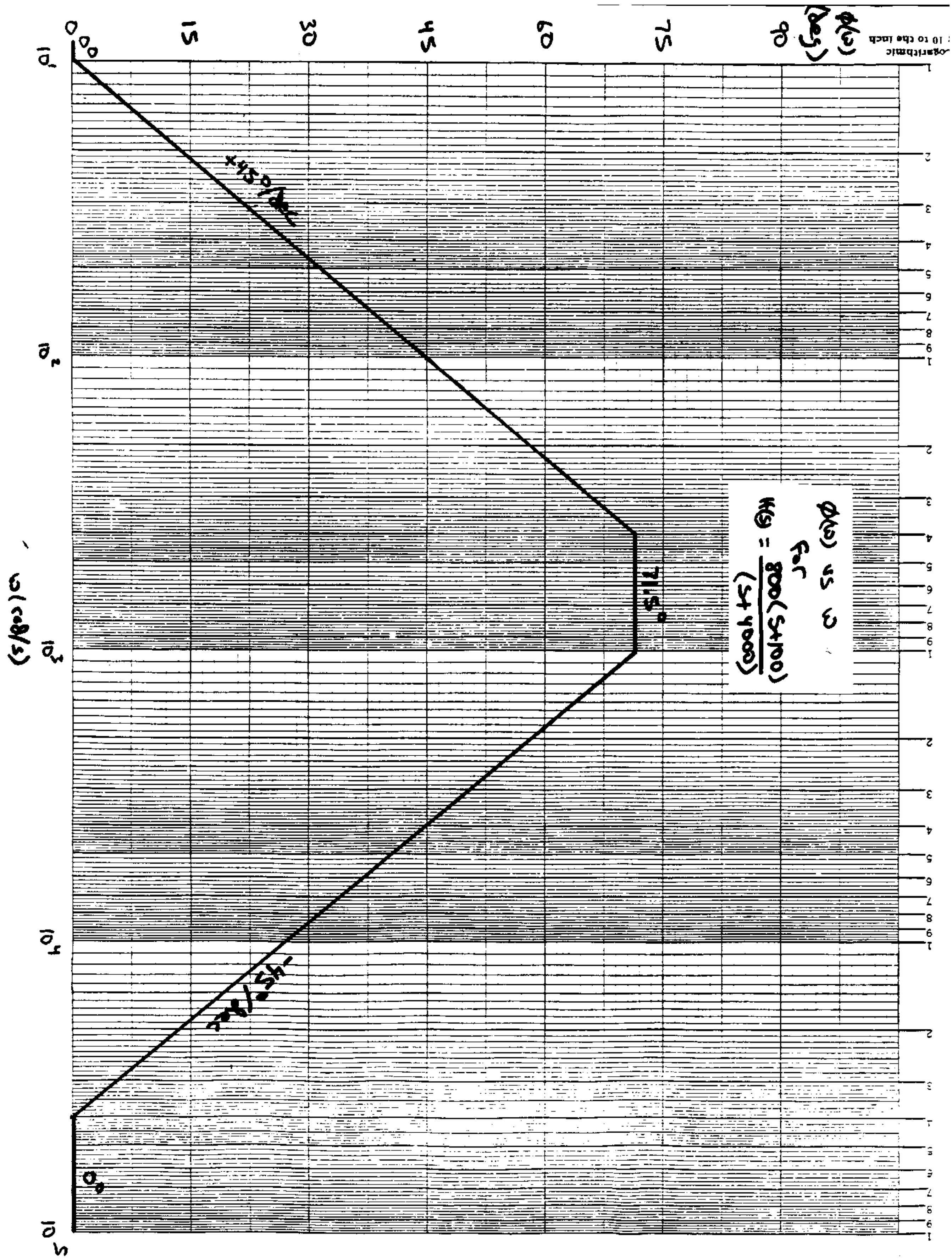
- Since there are no jw terms in the numerator or denominator of H(jw), the LM plot will begin level.
- The initial value will be 20log(20) = 26 dB
- The first break occurs at w = 100 (a zero), so the LM will break upward with a slope of +20 db/dec
- The second break occurs at w = 4000 (a pole), canceling the previous upward break, so the LM will level out.
- The Bode LM plot is shown on page 3.

Bode phase plot:

- Since there are no jw terms in the numerator or denominator of H(jw), the phase plot will begin at 0°.
- The zero at w = 100 will cause the phase to increase by 90° over a two decade range from 10 to 1,000 (slope of 45°/decade).
- The pole at w = 4,000 will cause the phase to decrease by 90° over a two decade range from 400 to 40,000 (slope of -45°/decade).
- Between w = 400 and w = 1000 the increase due to the zero and the decrease due to the pole will cancel, leaving the phase response level.
- After w = 40,000 the effect due to all poles and zeros is complete so the graph levels out at 0°. We could have easily predicted a final phase of 0° since there were equal numbers of poles and zeros and the 90° contribution due to each one effectively canceled one another).
- The Bode phase plot is shown on page 4.



LM(dB)
25
30
35
40
45
50
55
60
10^1
10^2
10^3
4000
10^4
10^5
ω(rad/s)
26 dB
+20 dB/dec
58 dB
LM vs ω for $H(s) = \frac{800(s+100)}{(s+4000)}$


φ(ω) (deg)
0
15
30
45
60
75
90
ω (rad/s)
10^1
10^2
10^3
10^4
10^5
+45°/dec
-45°/dec
71.5°
0°
0°
φ(ω) vs ω for H(s) = 800(s+100)/(s+4000)

B) Plotting Frequency Response (LM and Phase) using Excel

File: FreqResp

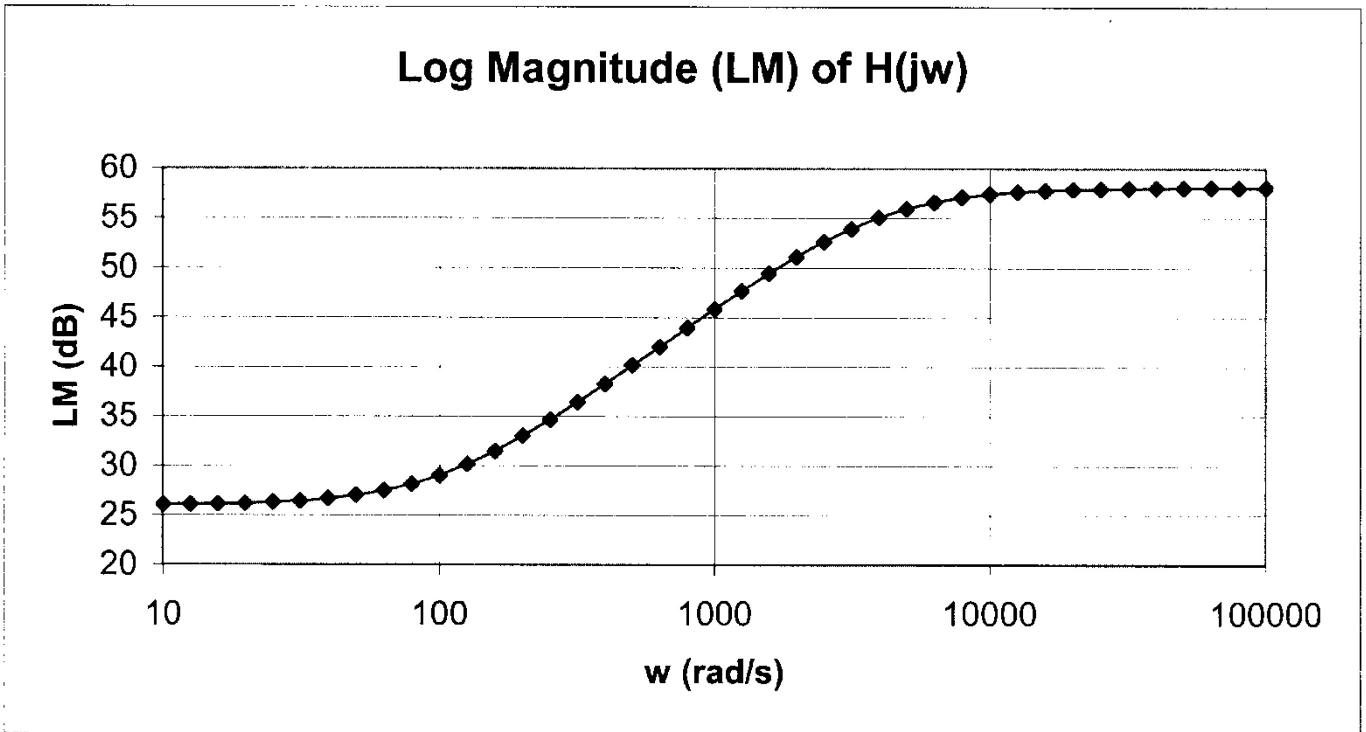
Problem: Use Excel to generate the exact LM and phase plots for the function below.

$$H(s) = \frac{800(s+100)}{(s+4000)}$$

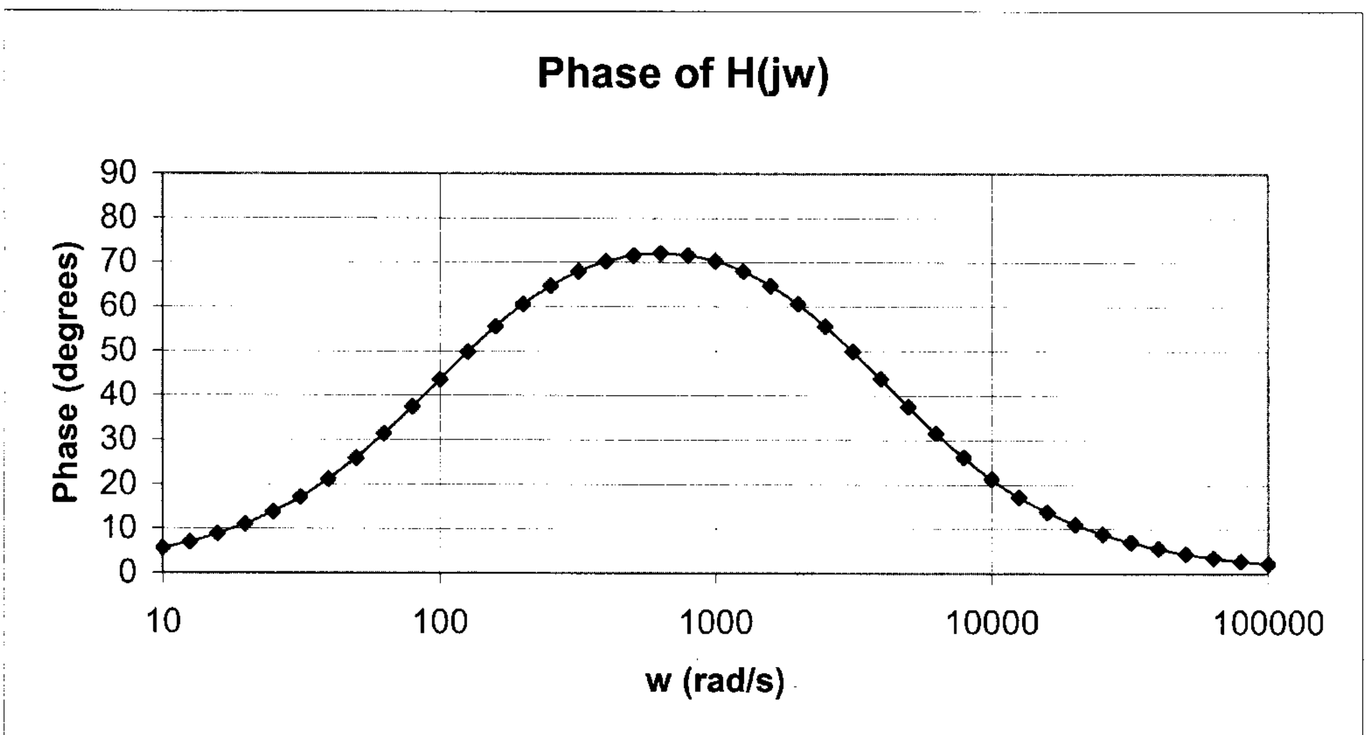
N	w (rad/s)	LM (dB)	Phase (degrees)
1.0	10	26.1	5.6
1.1	13	26.1	7.0
1.2	16	26.1	8.8
1.3	20	26.2	11.0
1.4	25	26.3	13.7
1.5	32	26.4	17.1
1.6	40	26.7	21.1
1.7	50	27.0	25.9
1.8	63	27.5	31.3
1.9	79	28.1	37.3
2.0	100	29.0	43.6
2.1	126	30.1	49.7
2.2	158	31.5	55.5
2.3	200	33.0	60.5
2.4	251	34.6	64.7
2.5	316	36.4	67.9
2.6	398	38.2	70.2
2.7	501	40.1	71.6
2.8	631	42.0	72.0
2.9	794	43.9	71.6
3.0	1000	45.8	70.3
3.1	1259	47.6	68.0
3.2	1585	49.4	64.8
3.3	1995	51.1	60.6
3.4	2512	52.6	55.6
3.5	3162	53.9	49.9
3.6	3981	55.0	43.7
3.7	5012	55.9	37.5
3.8	6310	56.6	31.5
3.9	7943	57.1	26.0
4.0	10000	57.4	21.2
4.1	12589	57.6	17.2
4.2	15849	57.8	13.8
4.3	19953	57.9	11.0
4.4	25119	58.0	8.8
4.5	31623	58.0	7.0
4.6	39811	58.0	5.6
4.7	50119	58.0	4.4
4.8	63096	58.0	3.5
4.9	79433	58.1	2.8
5.0	100000	58.1	2.2

Excel formulas:

Cell A13: 1.0
Cell B13: =10^A13
Cell C13: =20*LOG(800*SQRT(B13^2+100^2)/SQRT(B13^2+4000^2))
Cell D13: =DEGREES(ATAN(B13/100)-ATAN(B13/4000))



Log Magnitude (LM) of H(jw)
LM (dB)
60
55
50
45
40
35
30
25
20
10
100
1000
10000
100000
w (rad/s)



Phase of H(jw)
Phase (degrees)
90
80
70
60
50
40
30
20
10
0
10
100
1000
10000
100000
w (rad/s)

B) Plotting Frequency Response (LM and Phase) using MatLab

```
% EGR 261 - Signals & Systems
% Frequency Response (LM and phase) using MatLab
% Filename: FreqResp.m
% Problem:  Use MatLab to graph the log-magnitude (LM) and phase plots for
%       H(s) = 800(s+100)/(s+4000)
% Vary w from 10 rad/s to 100,000 rad/s
w = logspace(1,5,201);  % w=10^1 to 10^5 using 201 points (50 points/dec)
LM = 20*log10(800*sqrt(w.^2 + 100^2)./sqrt(w.^2 + 4000^2));
semilogx(w,LM,'k*-')
grid
title('LM vs w for H(s) = 800(s+100)/(s+4000)')
xlabel('radian frequency, w [rad/s]')
ylabel('Log-magnitude, LM [dB]')
Phase = atand(w/100) - atand(w/4000);  %Note: atand(x) yields angles in deg
figure     %Create a second graph window
semilogx(w,Phase,'k*-')
grid
title('Phase vs w for H(s) = 800(s+100)/(s+4000)')
xlabel('radian frequency, w [rad/s]')
ylabel('Phase [degrees]')
```

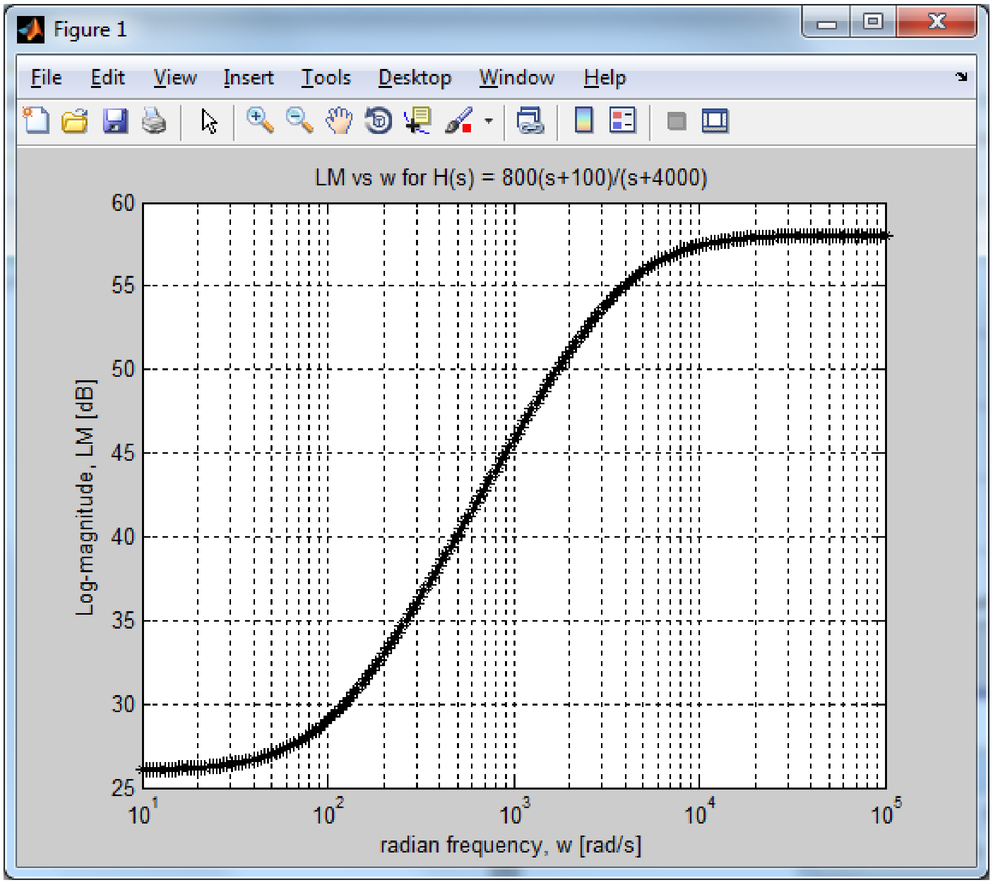


Figure 1
File Edit View Insert Tools Desktop Window Help
LM vs w for H(s) = 800(s+100)/(s+4000)
Log-magnitude, LM [dB]
60
55
50
45
40
35
30
25
10^1
10^2
10^3
10^4
10^5
radian frequency, w [rad/s]

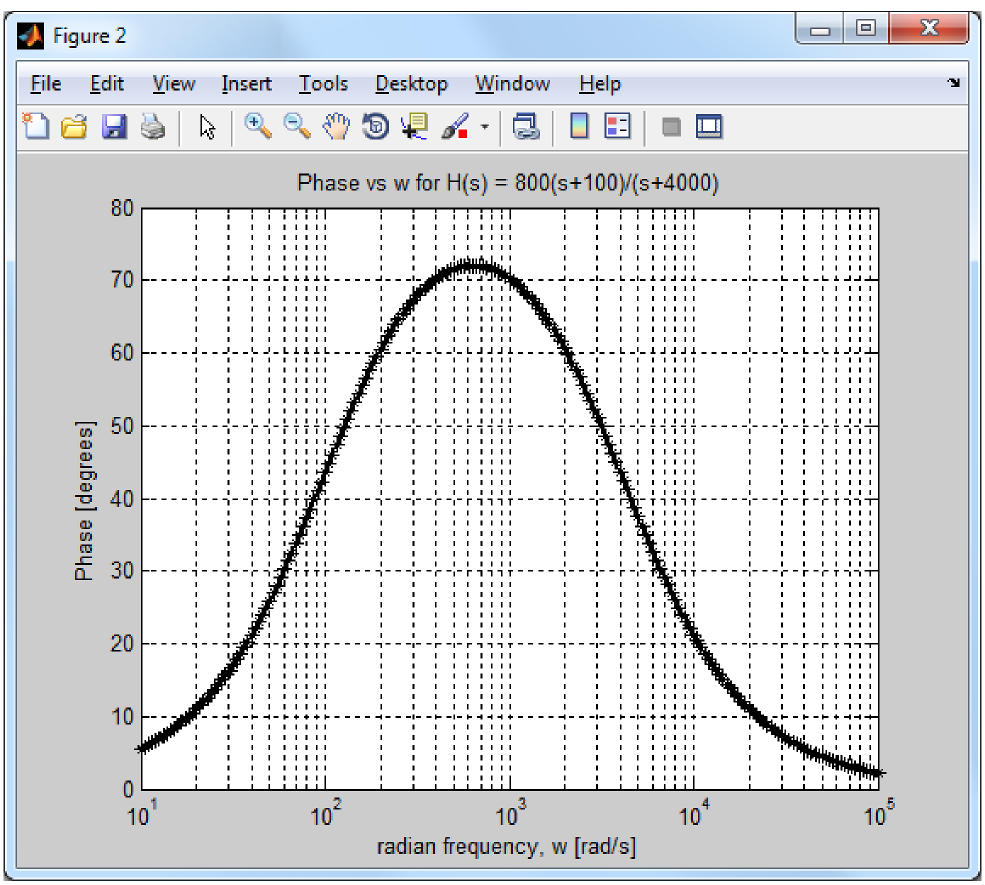


Figure 2
File Edit View Insert Tools Desktop Window Help
Phase vs w for H(s) = 800(s+100)/(s+4000)
Phase [degrees]
80
70
60
50
40
30
20
10
0
10^1
10^2
10^3
10^4
10^5
radian frequency, w [rad/s]

D) Plotting Frequency Response (LM and Phase) using MathCAD

File: FreqResp.MCD

Problem: Use MATHCAD to generate the exact LM and phase plots for the function

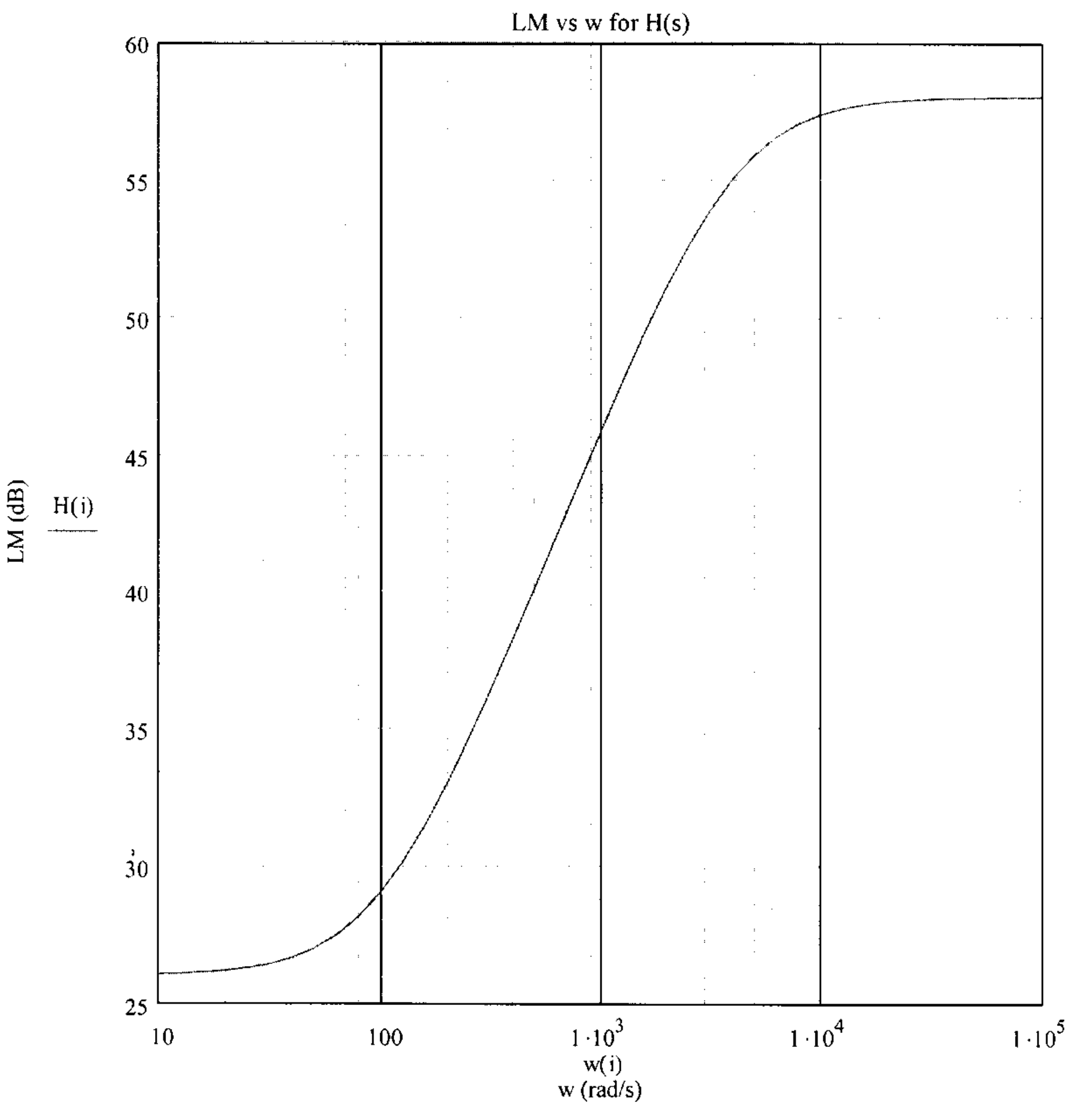
$$H(s) = \frac{800(s + 100)}{(s + 4{,}000)}$$

Solution:

$i := 1, 1.1 .. 5$ (this is a good method for specifying equally-spaced points on a log scale)

$w(i) := 10^i$ (the frequency w(i) will range from 10 rad/s to 100,000 rad/s)

$$H(i) := 20 \cdot \log\left(\frac{800 \cdot \sqrt{w(i)^2 + 100^2}}{\sqrt{w(i)^2 + 4000^2}}\right)$$ H(i) is the LM of H(s)



$$\phi rad(i) := atan\left(\frac{w(i)}{100}\right) - atan\left(\frac{w(i)}{4000}\right)$$ (phase of H(s) in radians)

$$\phi(i) := \phi rad(i)\cdot\frac{180}{\pi}$$ (convert the phase to degrees)