EGR 261
Signals and Systems
File: N261O1

Test #1 Overview

Material Covered:
Homework #1 and Homework #2 - Chapter 12 (Laplace transforms and inverse Laplace Transforms)
Homework #3 - Chapter 13 (Circuit analysis using Laplace transforms)
Sections 12.1-12.9, 13.1-13.5, 13.7 in Electric Circuits, 9th Edition, by Nilsson
Sections 4.1-4.4 in Linear Signals & Systems, 2nd Edition, by Lathi

Exam Materials:
- No calculators allowed
- Handout of Laplace Transform Properties and Common Laplace Transform Pairs provided (attached)
- No other formulas, books, etc allowed on the exam

Laplace Transforms and Inverse Laplace Transforms:
- Finding Laplace transforms:
  - By definition (only for simple functions)
  - No questions on convergence
  - Using known transforms and properties (primary method for finding F(s))
  - Find F(s) if a graph of f(t) is given
- Finding Inverse Laplace Transforms
  - Partial Fractions Expansion (PFE)
  - Use either of two methods to find PFE coefficients (your choice):
    - residue method
    - common denominator method
  - Be sure that order of N(s) < order of D(s) before using PFE – if not use long division
  - Complex roots – can be handled in two ways:
    - quadratic factors – put complex denominator in the form $(s + a)^2 + w^2$ – easiest method without a calculator
    - complex linear roots - simplify as much as possible if not using a calculator
- Trigonometric identities provided if necessary.
- No problems on solving differential equations using Laplace Transforms

Circuit Analysis using Laplace Transforms
- Find initial conditions ($v_C(0)$ and $i_L(0)$ only) if not provided
- Draw the Laplace Transformed Circuit (know the models for each component)
- Use basic circuit analysis techniques – your choice of method generally
- Partial solution may be specified – such as find V(s) or I(s) – in simplified polynomial form

Transfer functions:
- Finding transfer functions (H(s) = Y(s)/X(s)) for a given circuit
- Recall that transfer functions are always defined with zero initial conditions (so no sources in the models for L and C)
- Finding an output using a given transfer function – Y(s) = H(s)X(s) so $y(t) = \mathcal{L}^{-1}\{H(s)X(s)\}$
- Finding the impulse response for a given circuit or transfer function: impulse response = $h(t) = \mathcal{L}^{-1}\{H(s)\}$
- Finding the unit step response for a given circuit or transfer function: unit step response = $y(t) = \mathcal{L}^{-1}\{H(s)/s\}$

LAPLACE TRANSFORM PROPERTIES

1. **Linearity**: $\mathcal{L}\{af(t)\} = aF(s)$
2. **Superposition**: $\mathcal{L}\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$
3. **Modulation**: $\mathcal{L}\{e^{-at}f(t)\} = F(s + a)$
4. **Time-Shifting**: $\mathcal{L}\{f(t - \tau)u(t - \tau)\} = e^{-s\tau}F(s)$
5. **Scaling**: $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$
6. **Real Differentiation**: $\mathcal{L}\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0)$
7. **Real Integration**: $\mathcal{L}\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}F(s)$
8. **Complex Differentiation**: $\mathcal{L}\{tf(t)\} = -\frac{d}{ds}F(s)$
9. **Complex Integration**: $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)ds$
10. **Convolution**: $\mathcal{L}\{f(t) * g(t)\} = F(s) \cdot G(s)$

COMMON LAPLACE TRANSFORM PAIRS

f(t)	F(s)
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$
$\cos(wt)u(t)$	$\frac{s}{s^2 + w^2}$
$\sin(wt)u(t)$	$\frac{w}{s^2 + w^2}$
$e^{-at}\cos(wt)u(t)$	$\frac{s+a}{(s+a)^2 + w^2}$
$e^{-at}\sin(wt)u(t)$	$\frac{w}{(s+a)^2 + w^2}$
$2B\, e^{-at}\cos(wt+\theta)u(t)$	$\frac{\overline{B}}{s+a-jw} + \frac{\overline{B}^*}{s+a+jw}$, where $\overline{B} = B\angle\theta$