

## Homework Assignment #6 – Frequency Response

### Reading Assignment:

Read the following sections in Electric Circuits, 9<sup>th</sup> Edition by Nilsson:  
 Chapter 14, Chapter 15 (Sect. 1-2), Appendix D (The Decibel), and Appendix E (Bode Diagrams)  
 Section 14.9 in Linear Signals & Systems, 2nd Ed. by Lathi

### Problem Assignment:

- 1) Work the following Ch. 14 problems: 3, 4(a-b only), 7, 10(a-b only), 19, 22, 25  
 Work the following Ch. 15 problems: 2, 3, 22, 24  
 Note: Use  $R = 1 \text{ k}\Omega$  for 14.7E
- 2) Plot the Bode LM plots for the transfer functions below with to the following specifications:
  - use 4 cycle semi-log graph paper (available from instructor's web page)
  - label all levels in dB and all slopes in dB/decade

$$G_1(s) = \frac{500(s + 200)}{(s + 4,000)}$$

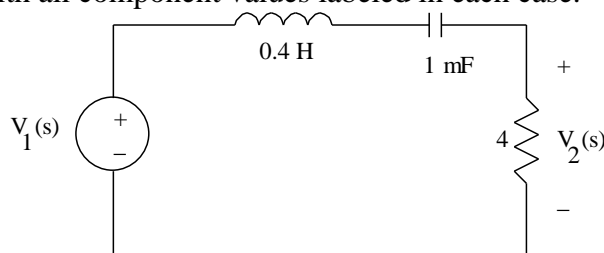
$$G_2(s) = \frac{500,000}{(s + 200)(s + 1000)}$$

$$G_3(s) = \frac{50(s + 2,000)(s + 5,000)}{(s + 10,000)(s + 80,000)}$$

$$G_4(s) = \frac{5,000(s)(s + 800)}{(s + 200)(s + 4,000)}$$

$$G_5(s) = \frac{0.5(s + 20)^2(s + 500)}{s(s + 100)}$$

- 3) Plot the Bode phase plot for  $G_1(s)$  and  $G_2(s)$  above with the following specifications:
  - use 4 cycle semi-log graph paper
  - label all levels in degrees and all slopes in degrees/decade
- 4) Plot the **exact** LM and phase plots for  $G_1(s)$  and  $G_2(s)$  using MATLAB, Excel, or some other graphing software. Include a printout of both the graph and the spreadsheet, equations, etc., used to produce the graph. Compare the graphs to the Bode plots generated for  $G_1(s)$  and  $G_2(s)$  in problems 2 and 3.
- 5) For the circuit shown below,  $H(s) = V_2(s)/V_1(s)$  is the transfer function of a bandpass filter with a center frequency of  $\omega_o = 50 \text{ rad/s}$ . Use impedance and frequency scaling on the circuit such that  $\omega_o = 20,000 \text{ rad/s}$  if
  - a)  $C = 1 \mu\text{F}$
  - b)  $L = 1 \text{ mH}$
  - c)  $R = 1 \text{ k}\Omega$
 Draw the scaled circuit with all component values labeled in each case.



### MATLAB Problems (Extra Credit):

Put all problems in a single MATLAB program with an initial block of comments (name, course assignment number, etc) and add a comment identifying each part. Include a printout of the program and the output. Check your results to make sure that they are correct.

- 1) Plot the **exact** LM and phase plots for  $G_3(s)$ ,  $G_4(s)$  and  $G_5(s)$  using MATLAB. Use the  $abs()$  and  $angle()$  functions in MATLAB, not  $tf()$  and  $bode()$ . Include a printout of both the graph and the spreadsheet, equations, etc., used to produce the graph. Compare the graphs to the Bode plots generated by hand in problems 2 and 3.
- 2) Repeat the previous step except use the  $tf()$  and  $bode()$  functions in MATLAB, not  $abs()$  and  $angle()$ . The results should match with the previous step.

### Selected Answers:

14.3) a) 
$$H(s) = \frac{\frac{R}{L}}{s + \frac{R + R_1}{L}}$$
 b)  $|H(j\omega)|_{\max}$  occurs when  $\omega = 0$

c)  $|H(j\omega)|_{\max} = \frac{R}{R + R_1}$  d)  $\omega_c = \frac{R + R_1}{L}$

e)  $\omega_c = 20,200$  rad/s  
 $H(j0) = 0.6287 / 0^\circ$   
 $H(j\omega_c) = H(j20,200) = 0.4446 / -45^\circ$   
 $H(j0.3\omega_c) = H(j6060) = 0.6022 / -16.70^\circ$   
 $H(j3\omega_c) = H(j60,600) = 0.1988 / -71.57^\circ$

14.4) a)  $\omega_c = 10,000$  rad/s,  $f_c = 1591.55$  Hz  
b) 
$$H(j\omega) = \frac{10,000}{10,000 + j\omega}$$
  
 $H(j\omega_c) = H(j10,000) = 0.7071 / -45^\circ$   
 $H(j0.1\omega_c) = H(j1000) = 0.9950 / -5.71^\circ$   
 $H(j10\omega_c) = H(j100,000) = 0.0995 / -84.29^\circ$

14.7) a) 
$$H(s) = \frac{\frac{1}{RC}}{s + \frac{R + R_L}{RR_L C}}$$
 b)  $|H(j\omega)|_{\max}$  occurs when  $\omega = 0$

c)  $|H(j\omega)|_{\max} = \frac{R_L}{R + R_L}$  d)  $\omega_c = \frac{1}{RC} \left( 1 + \frac{R}{R_L} \right)$

e)  $\omega_c = 11,000$  rad/s  
 $H(j0) = 0.9091 / 0^\circ$   
 $H(j\omega_c) = H(j11,000) = 0.6428 / -45^\circ$   
 $H(j0.1\omega_c) = H(j1,100) = 0.9046 / -5.71^\circ$   
 $H(j10\omega_c) = H(j110,000) = 0.0905 / -84.29^\circ$

14.10) a)  $f_c = 636.62 \text{ Hz}$

b)  $H(j\omega) = \frac{j\omega}{4000 + j\omega}$

$H(j\omega_c) = H(j4000) = 0.7071 / 45^\circ$

$H(j0.2\omega_c) = H(j800) = 0.1961 / 78.69^\circ$

$H(j5\omega_c) = H(j20,000) = 0.9806 / 11.31^\circ$

14.19)  $B = 1.99 \text{ kHz}, \quad f_{c1} = 9.01 \text{ kHz}, \quad f_{c2} = 7.02 \text{ kHz}$

14.22) a)  $\omega_o = 100 \text{ krad/s}$

b)  $f_o = 15.9 \text{ kHz}$

c)  $Q = 8$

d)  $\omega_{c1} = 93.95 \text{ krad/s}$

e)  $f_{c1} = 14.95 \text{ kHz}$

f)  $\omega_{c2} = 106.45 \text{ krad/s}$

g)  $f_{c2} = 16.94 \text{ kHz}$

h)  $B = 12.5 \text{ krad/s}$  or  $1.99 \text{ kHz}$

14.25) a)  $f_o = 3978.87 \text{ kHz}$

b)  $Q = 5$

c)  $f_{c1} = 3.60 \text{ kHz}$

d)  $f_{c2} = 4.40 \text{ kHz}$

e)  $B = 795.77 \text{ Hz}$

15.2) a)  $H(s) = \frac{-\left(\frac{C_1}{C_2}\right)\left(s + \frac{1}{R_1 C_1}\right)}{s + \frac{1}{R_2 C_2}}$       b)  $H(j0) = \frac{-R_2}{R_1}$       c)  $H(j\infty) = \frac{-C_1}{C_2}$

15.3) a)  $H(s) = \frac{-\left(\frac{1}{R_1 C_2}\right)(s)}{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}$       b)  $H(j0) = 0$       c)  $H(j\infty) = 0$

15.22)  $L' = 200 \text{ mH}, \quad i_o(t) = [10 - 10e^{-2500t} \cos(2500t)]u(t) \text{ mA}$

15.24)  $R' = 2546.48 \Omega, \quad L' = 253.3 \text{ uH}, \quad \omega_{c1} = 1180.6 \text{ krad/s}, \quad \omega_{c2} = 1337.7 \text{ krad/s},$   
 $B = 157.1 \text{ krad/s}$

5) a)  $R = 10, \quad L = 2.5 \text{ mH}, \quad C = 1 \text{ }\mu\text{F}$

b)  $R = 4, \quad L = 2.5 \text{ mH}, \quad C = 2.5 \text{ }\mu\text{F}$

c)  $R = 1000, \quad L = 250 \text{ mH}, \quad C = 10 \text{ nF}$