

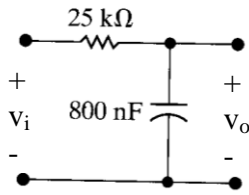
Homework Assignment #5 – Analysis of Systems

Reading Assignment:

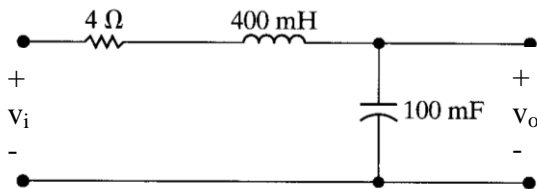
Ch. 2, Sect. 1-8 in *Linear Signals & Systems, 2nd Ed.* by Lathi
 Ch. 13, Sect. 6 in *Electric Circuits, 9th Ed.* by Nilsson

Problem Assignment:

1) For the circuit shown below:



- A) Determine the differential equation that represents the system if $v_i(t)$ is the input voltage and $v_o(t)$ is the output voltage.
 - B) Find the unit step response, USR, using the differential equation. Also sketch the USR.
 - C) Find the impulse response, $h(t)$, from the USR using $h(t) = d/dt(\text{USR})$. Also sketch $h(t)$.
 - D) Find the transfer function $H(s) = V_o(s)/V_i(s)$ from the DE in part A
 - E) Find the unit step response, USR, from $H(s)$
 - F) Find the impulse response, $h(t)$, from $H(s)$
- 2) Repeat problem 1 for the circuit shown below:



Note: For problems 3, 4, 5, and 6 below use a graphical approach in evaluating the convolution integral. Include sketches to illustrate each part of the solution as well as the related calculations. Sketch the final result as well.

- 3) Problem 2.4-18, parts a,c,f, and g in the Lathi text (Hint: Invert x_1 rather than x_2 on part g.)
- 4) Problem 13.60b in the Nilsson text
- 5) Problem 13.67 in the Nilsson text
- 6) Problem 13.62 in the Nilsson text
- 7) Repeat problem 13.62 using Laplace transforms. Specifically, find $H(s) = V_o(s)/V_i(s)$ and then find $v_o(t) = \mathcal{L}^{-1}\{H(s)V_i(s)\}$.

Selected Answers:

1A) $\frac{dV_o(t)}{dt} + 50V_o(t) = 50V_i(t)$

1B, E) USR = $V_o(t) = (1 - e^{-50t})u(t)$

1C, F) $h(t) = 50e^{-50t}u(t)$

1D) $H(s) = \frac{50}{s + 50}$

$$2A) \frac{d^2V_o(t)}{dt^2} + 10 \frac{dV_o(t)}{dt} + 25V_o(t) = 25V_i(t)$$

$$2B, E) \text{USR} = V_o(t) = (1 - 5te^{-5t} - e^{-5t})u(t)$$

$$2C, F) h(t) = 25te^{-5t}u(t)$$

$$2D) H(s) = \frac{25}{(s+5)^2}$$

$$2.4-18a) c(t) = \begin{cases} 0 & t \leq -1 \\ AB(t+1) & -1 \leq t \leq 0 \\ AB & 0 \leq t \leq 1 \\ AB(2-t) & 1 \leq t \leq 2 \\ 0 & 2 \leq t \end{cases}$$

$$2.4-18c) c(t) = \begin{cases} 0 & t \leq -4 \\ t+4 & -4 \leq t \leq -1 \\ 3 & t \geq -1 \end{cases}$$

$$2.4-18f) c(t) = \begin{cases} 0 & t \leq 0 \\ 1 - e^{-t} & 0 \leq t \leq 3 \\ e^{-(t-3)} - e^{-t} & t \geq 3 \end{cases}$$

$$2.4-18g) c(t) = \begin{cases} 0 & t \leq -1 \\ 0.5(1-t)^2 & -1 \leq t \leq 0 \\ 0.5 & t \geq 0 \end{cases}$$

$$13.62) y(t) = \begin{cases} 0 & t \leq 0 \\ (1 - e^{-t}) & 0 \leq t \leq 1 \\ (e-1)e^{-t} & t \geq 1 \end{cases}$$

$$13.60b) y(t) = \begin{cases} 0 & t \leq 0 \\ 312.5t & 0 \leq t \leq 10 \\ 3125 & 10 \leq t \leq 20 \\ 312.5(30-t) & 20 \leq t \leq 30 \\ 0 & t \geq 30 \end{cases}$$

$$13.67) v_o = \begin{cases} 0 & t \leq -1 \\ 5t^2 + 10t + 5 & -1 \leq t \leq 4 \\ 50t - 75 & 4 \leq t \leq 9 \\ -5t^2 + 140t - 480 & 9 \leq t \leq 14 \\ 500 & 14 \leq t \leq 19 \\ -5t^2 + 190t - 1305 & 19 \leq t \leq 24 \\ -50t + 1575 & 24 \leq t \leq 29 \\ 5t^2 - 340t + 5780 & 29 \leq t \leq 34 \\ 0 & 34 \leq t \end{cases}$$