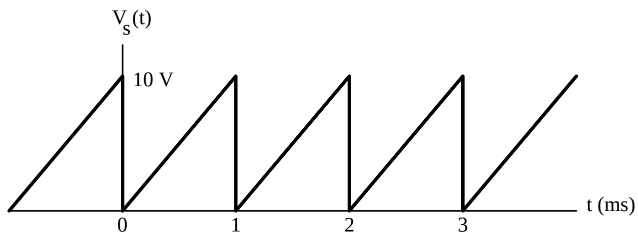
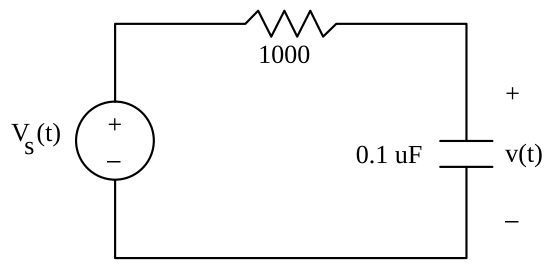
EGR 261
Signals and systems
File: Fourier Series – Circuit with Ramp Input

Analysis of a Series RC Circuit with a Ramp Wave Input

Problem: Determine the RMS value of the capacitor voltage, v(t), in the circuit shown below. The input voltage, $V_s(t)$, is an infinite periodic 0 to 10V ramp waveform.

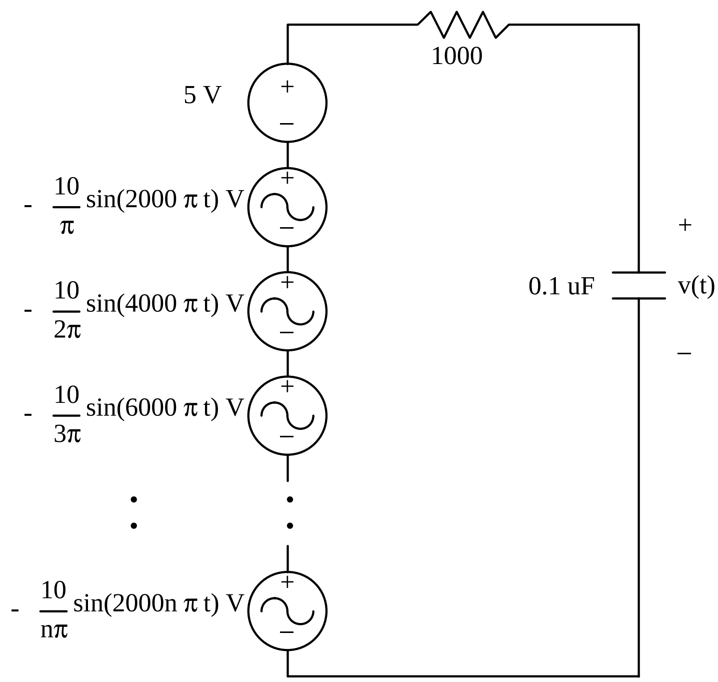




Solution: $V_s(t)$ should first be represented by a Fourier series. The Fourier series shown below was determined in a previous problem.

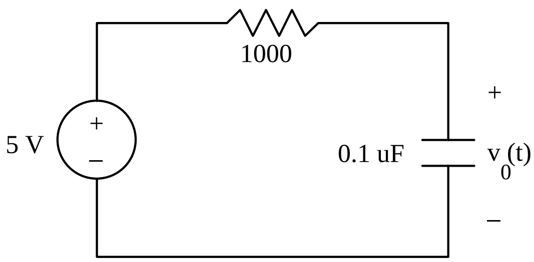
$$V_s(t) = 5 - \frac{10}{\pi}\sin(2000\pi t) - \frac{10}{2\pi}\sin(4000\pi t) - \frac{10}{3\pi}\sin(6000\pi t) - \frac{10}{4\pi}\sin(8000\pi t) - \bullet\bullet\bullet$$

We can now replace the ramp voltage source by an infinite number of sinusoidal voltage sources as shown below:

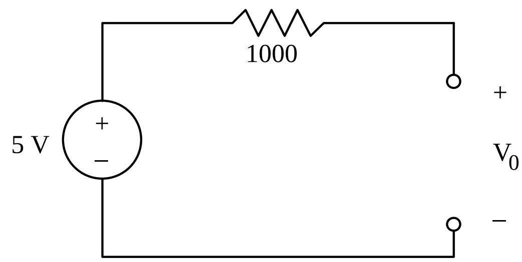


Now using superposition the circuit can be analyzed once for each source with all other sources killed (shorted). Note that the capacitor impedance changes for each circuit.

DC source only:

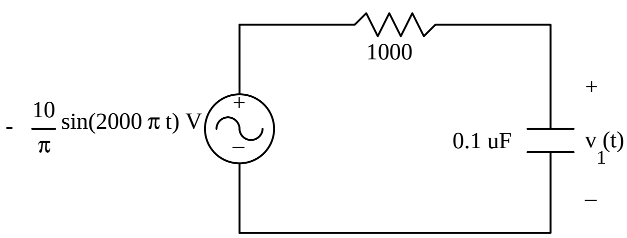


Equivalent Circuit:

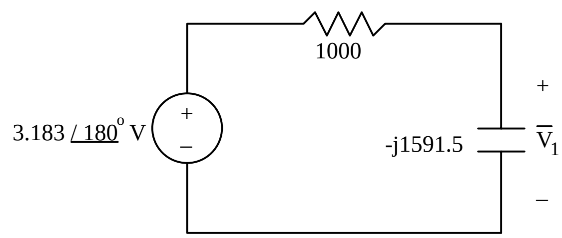


$\boxed{V_0 \;=\; 5\text{ V}}$ (DC component)

First harmonic source only:



AC Equivalent Circuit:

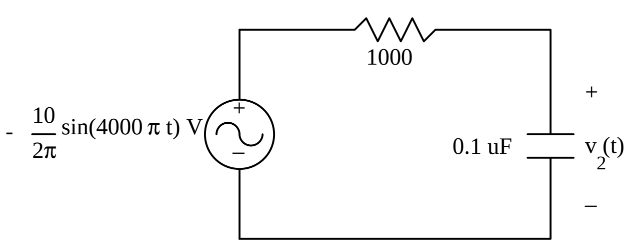


$$\overline{V}_1 \;=\; 3.183\;\underline{/180^\circ}\cdot\left[\frac{-j1591.5}{1000\;-\;j1591.5}\right]$$

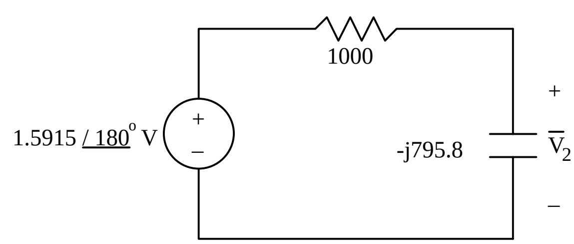
$$\overline{V}_1 \;=\; 2.695\;\underline{/147.9^\circ}$$

$\boxed{V_1 \;=\; 2.695\text{ V}}$ (1^{st} harmonic component)

Second harmonic source only:



AC Equivalent Circuit:

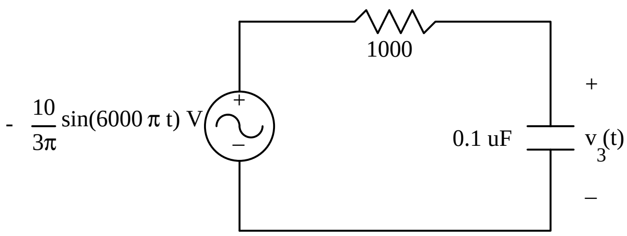


$$\overline{V}_2 \;=\; 1.5915\;\underline{/180^\circ}\cdot\left[\frac{-j795.8}{1000\;-\;\;-\,j795.8}\right]$$

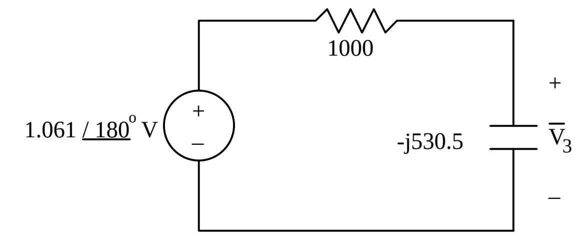
$$\overline{V}_2 \;=\; 0.991\;\underline{/128.5^\circ}$$

$\boxed{V_2 \;=\; 0.991\text{ V}}$ (2^{nd} harmonic component)

Third harmonic source only:



AC Equivalent Circuit:

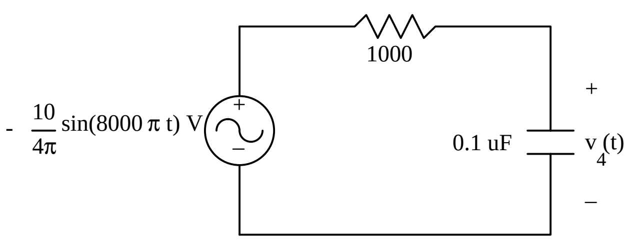


$$\bar{V}_3 = 1.061 \underline{/180^\circ} \cdot \left[\frac{-j530.5}{1000 \;-\; j530.5} \right]$$

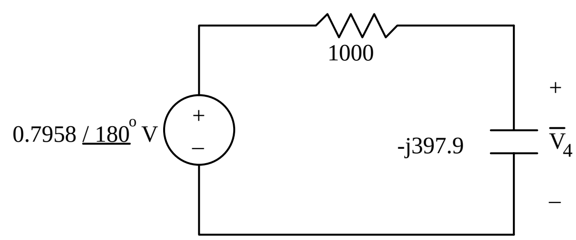
$$\bar{V}_3 = 0.4972 \underline{/117.9^\circ}$$

$\boxed{V_3 = 0.4972 \text{ V}}$ (3rd harmonic component)

Fourth harmonic source only:



AC Equivalent Circuit:

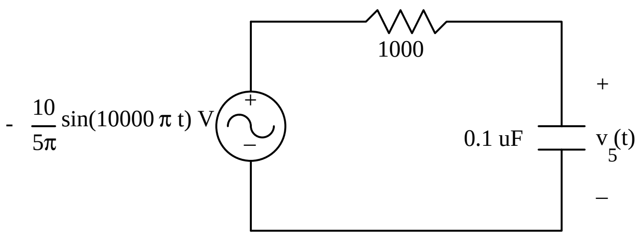


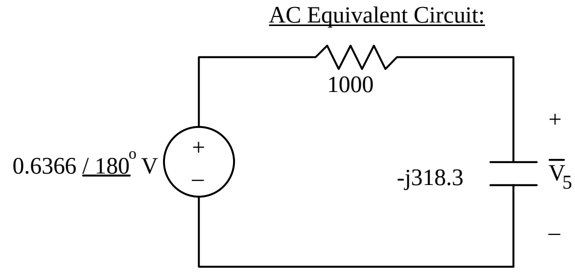
$$\bar{V}_4 = 0.7958 \underline{/180^\circ} \cdot \left[\frac{-j397.9}{1000 \;-\; j397.9} \right]$$

$$\bar{V}_4 = 0.2942 \underline{/111.6^\circ}$$

$\boxed{V_4 = 0.294 \text{ V}}$ (4th harmonic component)

Fifth harmonic source only:





$$\overline{V}_5 = 0.6366\ \underline{/180^\circ} \cdot \left[\frac{-j318.3}{1000\ -\ j318.3}\right]$$

$$\overline{V}_5 = 0.1931\ \underline{/107.6^\circ}$$

$\boxed{V_5 = 0.193\text{ V}}$ (5th harmonic component)

Total capacitor voltage:

In general, $V_{RMS} = \sqrt{V_0^2 + \dfrac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + \bullet\bullet\bullet}{2}}$ (using **peak** sinusoidal values)

In the example above, V_{RMS} can be approximated by using the DC component and the first 5 harmonics as follows:

$$V_{RMS} \approx \sqrt{5^2 + \frac{2.695^2 + 0.991^2 + 0.4972^2 + 0.2942^2 + 0.1931^2}{2}}$$

$\boxed{V_{RMS} \approx 5.414\text{ V}}$

On the following page MATHCAD is used to determine V_{RMS} by including 100 harmonics.