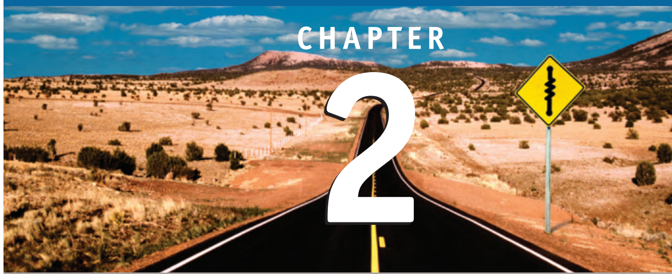

Circuit Elements

CHAPTER CONTENTS

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✓ CHAPTER OBJECTIVES

1. Understand the symbols for and the behavior of the following ideal basic circuit elements: independent voltage and current sources, dependent voltage and current sources, and resistors.
2. Be able to state Ohm's law, Kirchhoff's current law, and Kirchhoff's voltage law, and be able to use these laws to analyze simple circuits.
3. Know how to calculate the power for each element in a simple circuit and be able to determine whether or not the power balances for the whole circuit.

There are five ideal basic circuit elements: voltage sources, current sources, resistors, inductors, and capacitors. In this chapter we discuss the characteristics of voltage sources, current sources, and resistors. Although this may seem like a small number of elements with which to begin analyzing circuits, many practical systems can be modeled with just sources and resistors. They are also a useful starting point because of their relative simplicity; the mathematical relationships between voltage and current in sources and resistors are algebraic. Thus you will be able to begin learning the basic techniques of circuit analysis with only algebraic manipulations.

We will postpone introducing inductors and capacitors until Chapter 6, because their use requires that you solve integral and differential equations. However, the basic analytical techniques for solving circuits with inductors and capacitors are the same as those introduced in this chapter. So, by the time you need to begin manipulating more difficult equations, you should be very familiar with the methods of writing them.

Practical Perspective

Electrical Safety

"Danger—High Voltage." This commonly seen warning is misleading. All forms of energy, including electrical energy, can be hazardous. But it's not only the voltage that harms. The static electricity shock you receive when you walk across a carpet and touch a doorknob is annoying but does not injure. Yet that spark is caused by a voltage hundreds or thousands of times larger than the voltages that can cause harm.

The electrical energy that can actually cause injury is due to electrical current and how it flows through the body. Why, then, does the sign warn of high voltage? Because of the way electrical power is produced and distributed, it is easier to determine voltages than currents. Also, most electrical sources produce constant, specified voltages. So the signs warn about what is easy to measure. Determining whether and under what conditions a source can supply potentially dangerous currents is more difficult, as this requires an understanding of electrical engineering.

Before we can examine this aspect of electrical safety, we have to learn how voltages and currents are produced and the relationship between them. The electrical behavior of objects, such as the human body, is quite complex and often beyond complete comprehension. To allow us to predict and control electrical phenomena, we use simplifying models in which simple mathematical relationships between voltage and current approximate the actual relationships in real objects. Such models and analytical methods form the core of the electrical engineering techniques that will allow us to understand all electrical phenomena, including those relating to electrical safety.

At the end of this chapter, we will use a simple electric circuit model to describe how and why people are injured by electric currents. Even though we may never develop a complete and accurate explanation of the electrical behavior of the human body, we can obtain a close approximation using simple circuit models to assess and improve the safety of electrical systems and devices. Developing models that provide an understanding that is imperfect but adequate for solving practical problems lies at the heart of engineering. Much of the art of electrical engineering, which you will learn with experience, is in knowing when and how to solve difficult problems by using simplifying models.

2.1 Voltage and Current Sources

Before discussing ideal voltage and current sources, we need to consider the general nature of electrical sources. An **electrical source** is a device that is capable of converting nonelectric energy to electric energy and vice versa. A discharging battery converts chemical energy to electric energy, whereas a battery being charged converts electric energy to chemical energy. A dynamo is a machine that converts mechanical energy to electric energy and vice versa. If operating in the mechanical-to-electric mode, it is called a generator. If transforming from electric to mechanical energy, it is referred to as a motor. The important thing to remember about these sources is that they can either deliver or absorb electric power, generally maintaining either voltage or current. This behavior is of particular interest for circuit analysis and led to the creation of the ideal voltage source and the ideal current source as basic circuit elements. The challenge is to model practical sources in terms of the ideal basic circuit elements.

An **ideal voltage source** is a circuit element that maintains a prescribed voltage across its terminals regardless of the current flowing in those terminals. Similarly, an **ideal current source** is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals. These circuit elements do not exist as practical devices—they are idealized models of actual voltage and current sources.

Using an ideal model for current and voltage sources places an important restriction on how we may describe them mathematically. Because an ideal voltage source provides a steady voltage, even if the current in the element changes, it is impossible to specify the current in an ideal voltage source as a function of its voltage. Likewise, if the only information you have about an ideal current source is the value of current supplied, it is impossible to determine the voltage across that current source. We have sacrificed our ability to relate voltage and current in a practical source for the simplicity of using ideal sources in circuit analysis.

Ideal voltage and current sources can be further described as either independent sources or dependent sources. An **independent source** establishes a voltage or current in a circuit without relying on voltages or currents elsewhere in the circuit. The value of the voltage or current supplied is specified by the value of the independent source alone. In contrast, a **dependent source** establishes a voltage or current whose value depends on the value of a voltage or current elsewhere in the circuit. You cannot specify the value of a dependent source unless you know the value of the voltage or current on which it depends.

The circuit symbols for the ideal independent sources are shown in Fig. 2.1. Note that a circle is used to represent an independent source. To completely specify an ideal independent voltage source in a circuit, you must include the value of the supplied voltage and the reference polarity, as shown in Fig. 2.1(a). Similarly, to completely specify an ideal independent current source, you must include the value of the supplied current and its reference direction, as shown in Fig. 2.1(b).

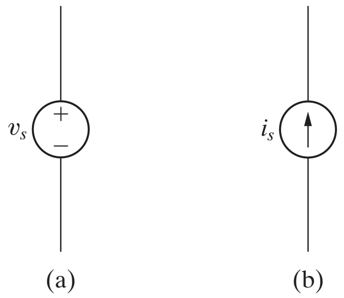


Figure 2.1 ▲ The circuit symbols for (a) an ideal independent voltage source and (b) an ideal independent current source.

The circuit symbols for the ideal dependent sources are shown in Fig. 2.2. A diamond is used to represent a dependent source. Both the dependent current source and the dependent voltage source may be controlled by either a voltage or a current elsewhere in the circuit, so there are a total of four variations, as indicated by the symbols in Fig. 2.2. Dependent sources are sometimes called **controlled sources**.

To completely specify an ideal dependent voltage-controlled voltage source, you must identify the controlling voltage, the equation that permits you to compute the supplied voltage from the controlling voltage, and the reference polarity for the supplied voltage. In Fig. 2.2(a), the controlling voltage is named v_x, the equation that determines the supplied voltage v_s is

$$v_s = \mu v_x,$$

and the reference polarity for v_s is as indicated. Note that μ is a multiplying constant that is dimensionless.

Similar requirements exist for completely specifying the other ideal dependent sources. In Fig. 2.2(b), the controlling current is i_x, the equation for the supplied voltage v_s is

$$v_s = \rho i_x,$$

the reference polarity is as shown, and the multiplying constant ρ has the dimension volts per ampere. In Fig. 2.2(c), the controlling voltage is v_x, the equation for the supplied current i_s is

$$i_s = \alpha v_x,$$

the reference direction is as shown, and the multiplying constant α has the dimension amperes per volt. In Fig. 2.2(d), the controlling current is i_x, the equation for the supplied current i_s is

$$i_s = \beta i_x,$$

the reference direction is as shown, and the multiplying constant β is dimensionless.

Finally, in our discussion of ideal sources, we note that they are examples of active circuit elements. An **active element** is one that models a device capable of generating electric energy. **Passive elements** model physical devices that cannot generate electric energy. Resistors, inductors, and capacitors are examples of passive circuit elements. Examples 2.1 and 2.2 illustrate how the characteristics of ideal independent and dependent sources limit the types of permissible interconnections of the sources.

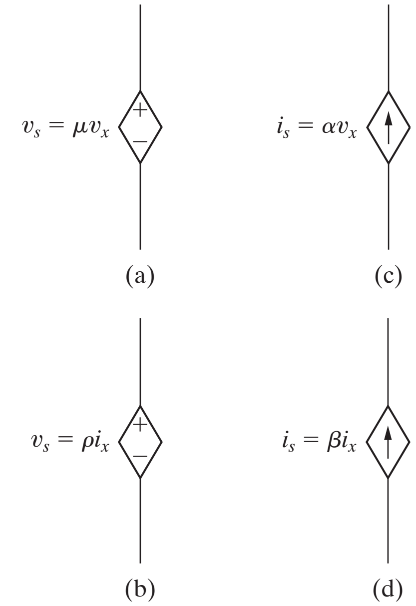


Figure 2.2 ▲ The circuit symbols for (a) an ideal dependent voltage-controlled voltage source, (b) an ideal dependent current-controlled voltage source, (c) an ideal dependent voltage-controlled current source, and (d) an ideal dependent current-controlled current source.

Example 2.1 Testing Interconnections of Ideal Sources

Using the definitions of the ideal independent voltage and current sources, state which interconnections in Fig. 2.3 are permissible and which violate the constraints imposed by the ideal sources.

Solution

Connection (a) is valid. Each source supplies voltage across the same pair of terminals, marked a,b. This requires that each source supply the same voltage with the same polarity, which they do.

Connection (b) is valid. Each source supplies current through the same pair of terminals, marked a,b. This requires that each source supply the same current in the same direction, which they do.

Connection (c) is not permissible. Each source supplies voltage across the same pair of terminals, marked a,b. This requires that each source supply the same voltage with the same polarity, which they do not.

Connection (d) is not permissible. Each source supplies current through the same pair of terminals, marked a,b. This requires that each source supply the same current in the same direction, which they do not.

Connection (e) is valid. The voltage source supplies voltage across the pair of terminals marked a,b. The current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same voltage regardless of the current, and an ideal current source supplies the same current regardless of the voltage, this is a permissible connection.

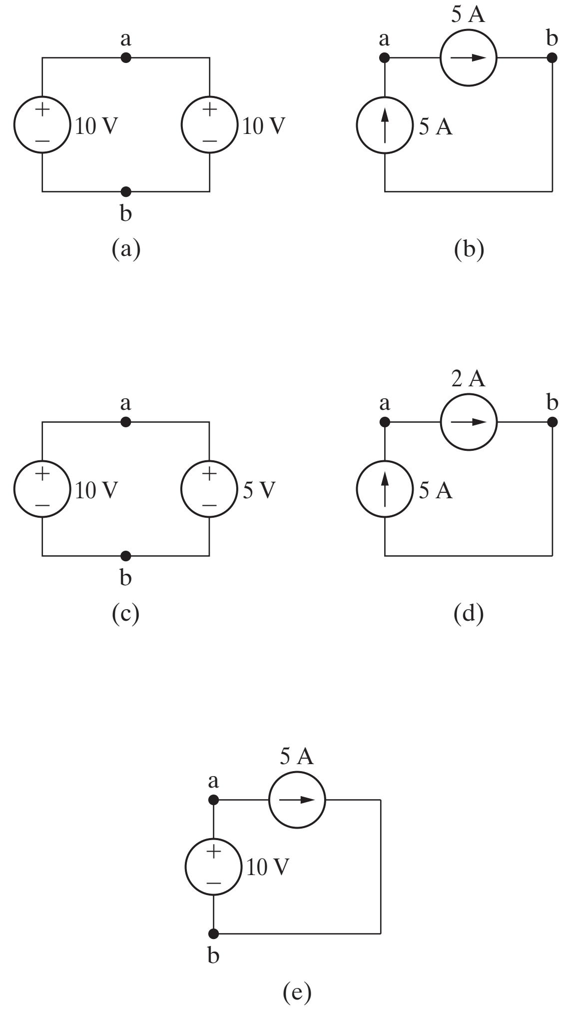


Figure 2.3 ▲ The circuits for Example 2.1.

Example 2.2 Testing Interconnections of Ideal Independent and Dependent Sources

Using the definitions of the ideal independent and dependent sources, state which interconnections in Fig. 2.4 are valid and which violate the constraints imposed by the ideal sources.

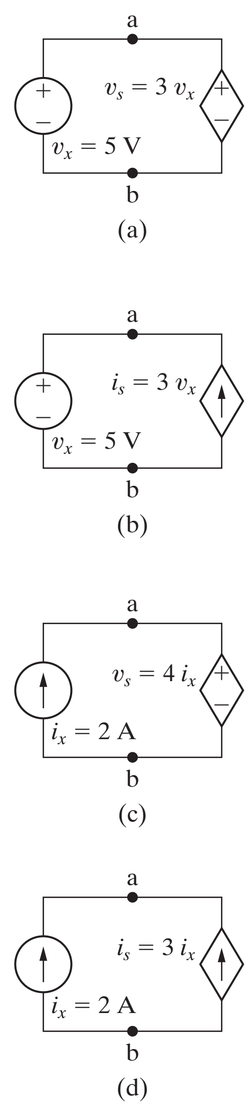


Figure 2.4 ▲ The circuits for Example 2.2.

Solution

Connection (a) is invalid. Both the independent source and the dependent source supply voltage across the same pair of terminals, labeled a,b. This requires that each source supply the same voltage with the same polarity. The independent source supplies 5 V, but the dependent source supplies 15 V.

Connection (b) is valid. The independent voltage source supplies voltage across the pair of terminals marked a,b. The dependent current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same voltage regardless of current, and an ideal current source supplies the same current regardless of voltage, this is an allowable connection.

Connection (c) is valid. The independent current source supplies current through the pair of terminals marked a,b. The dependent voltage source supplies voltage across the same pair of terminals. Because an ideal current source supplies the same current regardless of voltage, and an ideal voltage source supplies the same voltage regardless of current, this is an allowable connection.

Connection (d) is invalid. Both the independent source and the dependent source supply current through the same pair of terminals, labeled a,b. This requires that each source supply the same current in the same reference direction. The independent source supplies 2 A, but the dependent source supplies 6 A in the opposite direction.

✓ ASSESSMENT PROBLEMS

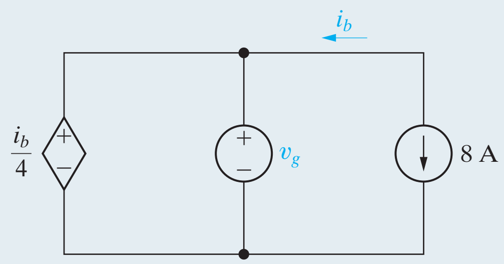
Objective 1—Understand ideal basic circuit elements

2.1 For the circuit shown,

a) What value of v_g is required in order for the interconnection to be valid?

b) For this value of v_g, find the power associated with the 8 A source.

Answer: (a) −2 V;
(b) −16 W (16 W delivered).

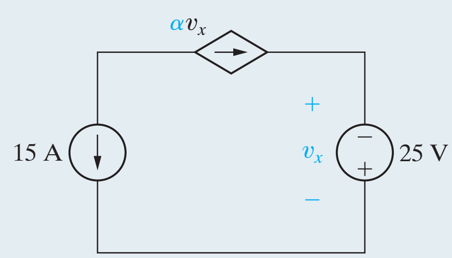


2.2 For the circuit shown,

a) What value of α is required in order for the interconnection to be valid?

b) For the value of α calculated in part (a), find the power associated with the 25 V source.

Answer: (a) 0.6 A/V;
(b) 375 W (375 W absorbed).



NOTE: Also try Chapter Problems 2.2 and 2.3.

2.2 Electrical Resistance (Ohm's Law)

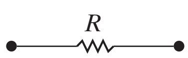


Figure 2.5 ▲ The circuit symbol for a resistor having a resistance R.

Resistance is the capacity of materials to impede the flow of current or, more specifically, the flow of electric charge. The circuit element used to model this behavior is the **resistor**. Figure 2.5 shows the circuit symbol for the resistor, with R denoting the resistance value of the resistor.

Conceptually, we can understand resistance if we think about the moving electrons that make up electric current interacting with and being resisted by the atomic structure of the material through which they are moving. In the course of these interactions, some amount of electric energy is converted to thermal energy and dissipated in the form of heat. This effect may be undesirable. However, many useful electrical devices take advantage of resistance heating, including stoves, toasters, irons, and space heaters.

Most materials exhibit measurable resistance to current. The amount of resistance depends on the material. Metals such as copper and aluminum have small values of resistance, making them good choices for wiring used to conduct electric current. In fact, when represented in a circuit diagram, copper or aluminum wiring isn't usually modeled as a resistor; the resistance of the wire is so small compared to the resistance of other elements in the circuit that we can neglect the wiring resistance to simplify the diagram.

For purposes of circuit analysis, we must reference the current in the resistor to the terminal voltage. We can do so in two ways: either in the direction of the voltage drop across the resistor or in the direction of the voltage rise across the resistor, as shown in Fig. 2.6. If we choose the former, the relationship between the voltage and current is

$$v = iR, \tag{2.1}$$

◀ **Ohm's law**

where

v = the voltage in volts,

i = the current in amperes,

R = the resistance in ohms.

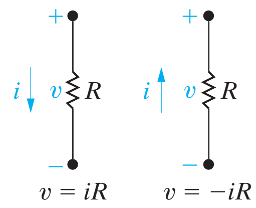


Figure 2.6 ▲ Two possible reference choices for the current and voltage at the terminals of a resistor, and the resulting equations.

If we choose the second method, we must write

$$v = -iR, \tag{2.2}$$

where v, i, and R are, as before, measured in volts, amperes, and ohms, respectively. The algebraic signs used in Eqs. 2.1 and 2.2 are a direct consequence of the passive sign convention, which we introduced in Chapter 1.

Equations 2.1 and 2.2 are known as **Ohm's law** after Georg Simon Ohm, a German physicist who established its validity early in the nineteenth century. Ohm's law is the algebraic relationship between voltage and current for a resistor. In SI units, resistance is measured in ohms. The Greek letter omega (Ω) is the standard symbol for an ohm. The circuit diagram symbol for an 8 Ω resistor is shown in Fig. 2.7.

Figure 2.7 ▲ The circuit symbol for an 8 Ω resistor.

Ohm's law expresses the voltage as a function of the current. However, expressing the current as a function of the voltage also is convenient. Thus, from Eq. 2.1,

$$i = \frac{v}{R}, \tag{2.3}$$

or, from Eq. 2.2,

$$i = -\frac{v}{R}. \tag{2.4}$$

The reciprocal of the resistance is referred to as **conductance**, is symbolized by the letter G, and is measured in siemens (S). Thus

$$G = \frac{1}{R} \text{ S}. \tag{2.5}$$

An 8 Ω resistor has a conductance value of 0.125 S. In much of the professional literature, the unit used for conductance is the mho (ohm spelled backward), which is symbolized by an inverted omega (℧). Therefore we may also describe an 8 Ω resistor as having a conductance of 0.125 mho, (℧).

We use ideal resistors in circuit analysis to model the behavior of physical devices. Using the qualifier *ideal* reminds us that the resistor model makes several simplifying assumptions about the behavior of actual resistive devices. The most important of these simplifying assumptions is that the resistance of the ideal resistor is constant and its value does not vary over time. Most actual resistive devices do not have constant resistance, and their resistance does vary over time. The ideal resistor model can be used to represent a physical device whose resistance doesn't vary much from some constant value over the time period of interest in the circuit analysis. In this book we assume that the simplifying assumptions about resistance devices are valid, and we thus use ideal resistors in circuit analysis.

We may calculate the power at the terminals of a resistor in several ways. The first approach is to use the defining equation and simply calculate the product of the terminal voltage and current. For the reference systems shown in Fig. 2.6, we write

$$p = vi \tag{2.6}$$

when $v = iR$ and

$$p = -vi \tag{2.7}$$

when $v = -iR$.

A second method of expressing the power at the terminals of a resistor expresses power in terms of the current and the resistance. Substituting Eq. 2.1 into Eq. 2.6, we obtain

$$p = vi = (iR)i$$

so

Power in a resistor in terms of current ▶

$$p = i^2 R. \tag{2.8}$$

Likewise, substituting Eq. 2.2 into Eq. 2.7, we have

$$p = -vi = -(-iR)i = i^2 R. \tag{2.9}$$

Equations 2.8 and 2.9 are identical and demonstrate clearly that, regardless of voltage polarity and current direction, the power at the terminals of a resistor is positive. Therefore, a resistor absorbs power from the circuit.

A third method of expressing the power at the terminals of a resistor is in terms of the voltage and resistance. The expression is independent of the polarity references, so

Power in a resistor in terms of voltage ▶

$$p = \frac{v^2}{R}. \tag{2.10}$$

Sometimes a resistor's value will be expressed as a conductance rather than as a resistance. Using the relationship between resistance and conductance given in Eq. 2.5, we may also write Eqs. 2.9 and 2.10 in terms of the conductance, or

$$p = \frac{i^2}{G}, \tag{2.11}$$

$$p = v^2 G. \tag{2.12}$$

Equations 2.6–2.12 provide a variety of methods for calculating the power absorbed by a resistor. Each yields the same answer. In analyzing a circuit, look at the information provided and choose the power equation that uses that information directly.

Example 2.3 illustrates the application of Ohm's law in conjunction with an ideal source and a resistor. Power calculations at the terminals of a resistor also are illustrated.

Example 2.3 Calculating Voltage, Current, and Power for a Simple Resistive Circuit

In each circuit in Fig. 2.8, either the value of v or i is not known.

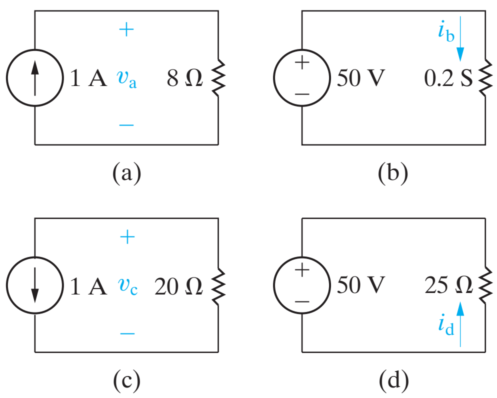


Figure 2.8 ▲ The circuits for Example 2.3.

a) Calculate the values of v and i.

b) Determine the power dissipated in each resistor.

Solution

a) The voltage v_a in Fig. 2.8(a) is a drop in the direction of the current in the resistor. Therefore,

$$v_a = (1)(8) = 8 \text{ V}.$$

The current i_b in the resistor with a conductance of 0.2 S in Fig. 2.8(b) is in the direction of the voltage drop across the resistor. Thus

$$i_b = (50)(0.2) = 10 \text{ A}.$$

The voltage v_c in Fig. 2.8(c) is a rise in the direction of the current in the resistor. Hence

$$v_c = -(1)(20) = -20 \text{ V}.$$

The current i_d in the 25 Ω resistor in Fig. 2.8(d) is in the direction of the voltage rise across the resistor. Therefore

$$i_d = \frac{-50}{25} = -2 \text{ A}.$$

b) The power dissipated in each of the four resistors is

$$p_{8\Omega} = \frac{(8)^2}{8} = (1)^2(8) = 8 \text{ W},$$

$$p_{0.2S} = (50)^2(0.2) = 500 \text{ W},$$

$$p_{20\Omega} = \frac{(-20)^2}{20} = (1)^2(20) = 20 \text{ W},$$

$$p_{25\Omega} = \frac{(50)^2}{25} = (-2)^2(25) = 100 \text{ W}.$$

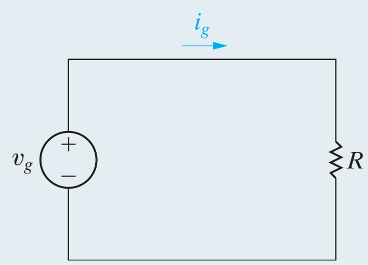
Having introduced the general characteristics of ideal sources and resistors, we next show how to use these elements to build the circuit model of a practical system.

✓ ASSESSMENT PROBLEMS

Objective 2—Be able to state and use Ohm's Law . . .

2.3 For the circuit shown,

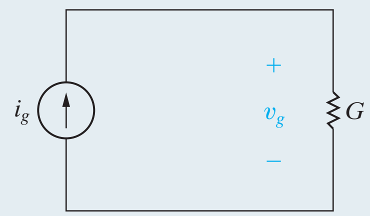
a) If $v_g = 1$ kV and $i_g = 5$ mA, find the value of R and the power absorbed by the resistor.
b) If $i_g = 75$ mA and the power delivered by the voltage source is 3 W, find v_g, R, and the power absorbed by the resistor.
c) If $R = 300\ \Omega$ and the power absorbed by R is 480 mW, find i_g and v_g.



Answer: (a) 200 kΩ, 5 W;
(b) 40 V, 533.33 Ω, 3 W;
(c) 40 mA, 12 V.

2.4 For the circuit shown,

a) If $i_g = 0.5$ A and $G = 50$ mS, find v_g and the power delivered by the current source.
b) If $v_g = 15$ V and the power delivered to the conductor is 9 W, find the conductance G and the source current i_g.
c) If $G = 200\ \mu$S and the power delivered to the conductance is 8 W, find i_g and v_g.



Answer: (a) 10 V, 5 W;
(b) 40 mS, 0.6 A;
(c) 40 mA, 200 V.

NOTE: Also try Chapter Problems 2.6 and 2.8.

2.3 Construction of a Circuit Model

We have already stated that one reason for an interest in the basic circuit elements is that they can be used to construct circuit models of practical systems. The skill required to develop a circuit model of a device or system is as complex as the skill required to solve the derived circuit. Although this text emphasizes the skills required to solve circuits, you also will need other skills in the practice of electrical engineering, and one of the most important is modeling.

We develop circuit models in the next two examples. In Example 2.4 we construct a circuit model based on a knowledge of the behavior of the system's components and how the components are interconnected. In Example 2.5 we create a circuit model by measuring the terminal behavior of a device.

Example 2.4 Constructing a Circuit Model of a Flashlight

Construct a circuit model of a flashlight.

Solution

We chose the flashlight to illustrate a practical system because its components are so familiar. Figure 2.9(a) shows a photograph of a widely available flashlight. Figure 2.9(b) shows the disassembled flashlight's components.

When a flashlight is regarded as an electrical system, the components of primary interest are the batteries, the lamp, the connector, the case, and the switch. We now consider the circuit model for each component.

A dry-cell battery maintains a reasonably constant terminal voltage if the current demand is not excessive. Thus if the dry-cell battery is operating within its intended limits, we can model it with an ideal voltage source. The prescribed voltage then is constant and equal to the sum of two dry-cell values.

The ultimate output of the lamp is light energy, which is achieved by heating the filament in the lamp to a temperature high enough to cause radiation in the visible range. We can model the lamp with an ideal resistor. Note in this case that although the resistor accounts for the amount of electric energy converted to thermal energy, it does not predict how much of the thermal energy is converted to light energy. The resistor used to represent the lamp does predict the steady current drain on the batteries, a characteristic of the system that also is of interest. In this model, R_l symbolizes the lamp resistance.

The connector used in the flashlight serves a dual role. First, it provides an electrical conductive path between the dry cells and the case. Second, it is formed into a springy coil so that it also can apply mechanical pressure to the contact between the batteries and the lamp. The purpose of this mechanical pressure is to maintain contact between the two dry cells and between the dry cells and the lamp. Hence, in choosing the wire for the connector, we may find that its mechanical properties are more important than its electrical properties for the flashlight design. Electrically, we can model the connector with an ideal resistor, labeled R_1.

The case also serves both a mechanical and an electrical purpose. Mechanically, it contains all the other components and provides a grip for the person using it. Electrically, it provides a connection between

Figure 2.9 ▲ The flashlight viewed as an electrical system. (a) Flashlight. (b) Disassembled flashlight.

other elements in the flashlight. If the case is metal, it conducts current between the batteries and the lamp. If it is plastic, a metal strip inside the case connects the coiled connector to the switch. Either way, an ideal resistor, which we denote R_c, models the electrical connection provided by the case.

The final component is the switch. Electrically, the switch is a two-state device. It is either ON or OFF. An ideal switch offers no resistance to the current when it is in the ON state, but it offers infinite resistance to current when it is in the OFF state. These two states represent the limiting values of a resistor; that is, the ON state corresponds to a resistor with a numerical value of zero, and the OFF state corresponds to a resistor with a numerical value of infinity. The two extreme values have the descriptive names **short circuit** ($R = 0$) and **open circuit** ($R = \infty$). Figure 2.10(a) and (b) show the graphical representation of a short circuit and an open circuit, respectively. The symbol shown in Fig. 2.10(c) represents the fact that a switch can be either a short circuit or an open circuit, depending on the position of its contacts.

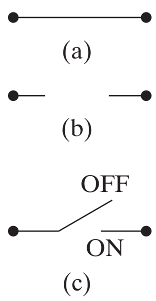


Figure 2.10 ▲ Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.

We now construct the circuit model of the flashlight. Starting with the dry-cell batteries, the positive terminal of the first cell is connected to the negative terminal of the second cell, as shown in Fig. 2.11. The positive terminal of the second cell is connected to one terminal of the lamp. The other terminal of the lamp makes contact with one side of the switch, and the other side of the switch is connected to the metal case. The metal case is then connected to the negative terminal of the first dry cell by means of the metal spring. Note that the elements form a closed path or circuit. You can see the closed path formed by the connected elements in Fig. 2.11. Figure 2.12 shows a circuit model for the flashlight.

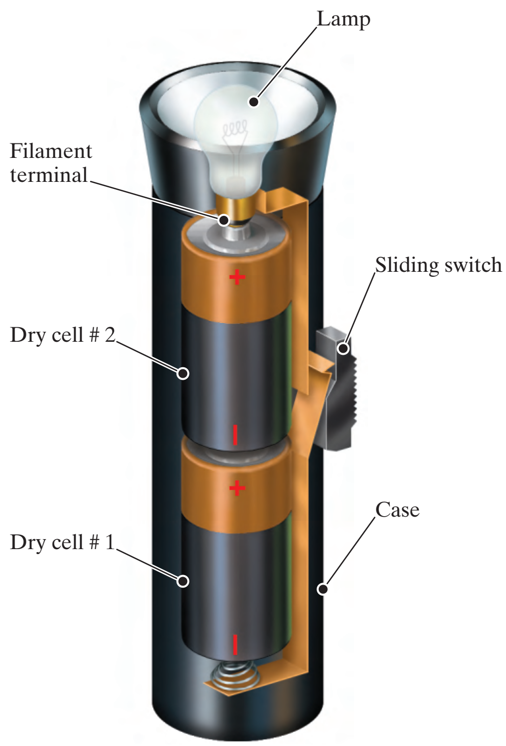


Figure 2.11 ▲ The arrangement of flashlight components.

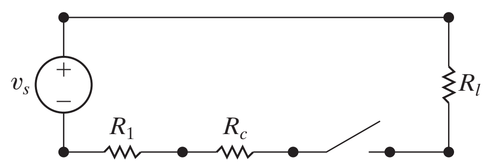


Figure 2.12 ▲ A circuit model for a flashlight.

We can make some general observations about modeling from our flashlight example: First, in developing a circuit model, the *electrical* behavior of each physical component is of primary interest. In the flashlight model, three very different physical components—a lamp, a coiled wire, and a metal case—are all represented by the same circuit element (a resistor), because the electrical phenomenon taking place in each is the same. Each is presenting resistance to the current flowing through the circuit.

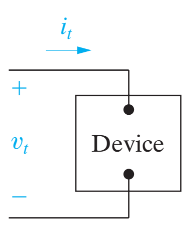
Second, circuit models may need to account for undesired as well as desired electrical effects. For example, the heat resulting from the resistance in the lamp produces the light, a desired effect. However, the heat resulting from the resistance in the case and coil represents an unwanted or parasitic effect. It drains the dry cells and produces no useful output. Such parasitic effects must be considered or the resulting model may not adequately represent the system.

And finally, modeling requires approximation. Even for the basic system represented by the flashlight, we made simplifying assumptions in developing the circuit model. For example, we assumed an ideal switch, but in practical switches, contact resistance may be high enough to interfere with proper operation of the system. Our model does not predict this behavior. We also assumed that the coiled connector exerts enough pressure to eliminate any contact resistance between the dry cells. Our model does not predict the effect of inadequate pressure. Our use of an ideal voltage source ignores any internal dissipation of energy in the dry cells, which might be due to the parasitic heating just mentioned. We could account for this by adding an ideal resistor between the source and the lamp resistor. Our model assumes the internal loss to be negligible.

In modeling the flashlight as a circuit, we had a basic understanding of and access to the internal components of the system. However, sometimes we know only the terminal behavior of a device and must use this information in constructing the model. Example 2.5 explores such a modeling problem.

Example 2.5 Constructing a Circuit Model Based on Terminal Measurements

The voltage and current are measured at the terminals of the device illustrated in Fig. 2.13(a), and the values of v_t and i_t are tabulated in Fig. 2.13(b). Construct a circuit model of the device inside the box.



(a)

v_t (V)	i_t (A)
−40	−10
−20	−5
0	0
20	5
40	10

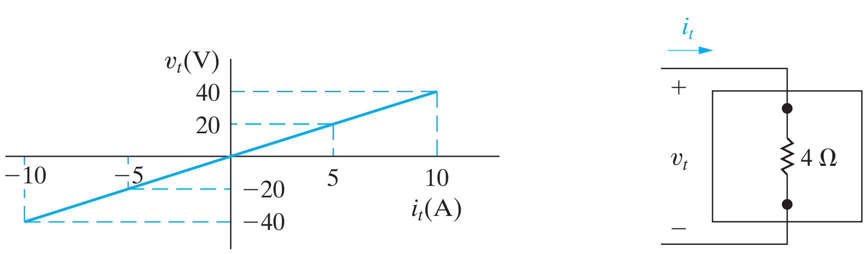
(b)

Figure 2.13 ▲ The (a) device and (b) data for Example 2.5.

Solution

Plotting the voltage as a function of the current yields the graph shown in Fig. 2.14(a). The equation of the line in this figure illustrates that the terminal voltage is directly proportional to the terminal current, $v_t = 4i_t$. In terms of Ohm's law, the device inside the box behaves like a 4 Ω resistor. Therefore, the circuit model for the device inside the box is a 4 Ω resistor, as seen in Fig. 2.14(b).

We come back to this technique of using terminal characteristics to construct a circuit model after introducing Kirchhoff's laws and circuit analysis.



(a) (b)

Figure 2.14 ▲ (a) The values of v_t versus i_t for the device in Fig. 2.13. (b) The circuit model for the device in Fig. 2.13.

NOTE: Assess your understanding of this example by trying Chapter Problems 2.10 and 2.11.

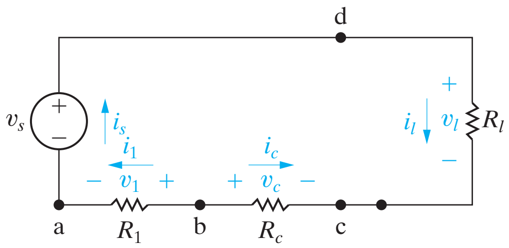


Figure 2.15 ▲ Circuit model of the flashlight with assigned voltage and current variables.

2.4 Kirchhoff's Laws

A circuit is said to be solved when the voltage across and the current in every element have been determined. Ohm's law is an important equation for deriving such solutions. However, Ohm's law may not be enough to provide a complete solution. As we shall see in trying to solve the flashlight circuit from Example 2.4, we need to use two more important algebraic relationships, known as Kirchhoff's laws, to solve most circuits.

We begin by redrawing the circuit as shown in Fig. 2.15, with the switch in the ON state. Note that we have also labeled the current and voltage variables associated with each resistor and the current associated with the voltage source. Labeling includes reference polarities, as always. For convenience, we attach the same subscript to the voltage and current labels as we do to the resistor labels. In Fig. 2.15, we also removed some of the terminal dots of Fig. 2.12 and have inserted nodes. Terminal dots are the start and end points of an individual circuit element. A **node** is a point where two or more circuit elements meet. It is necessary to identify nodes in order to use Kirchhoff's current law, as we will see in a moment. In Fig. 2.15, the nodes are labeled a, b, c, and d. Node d connects the battery and the lamp and in essence stretches all the way across the top of the diagram, though we label a single point for convenience. The dots on either side of the switch indicate its terminals, but only one is needed to represent a node, so only one is labeled node c.

For the circuit shown in Fig. 2.15, we can identify seven unknowns: i_s, i_1, i_c, i_l, v_1, v_c, and v_l. Recall that v_s is a known voltage, as it represents the sum of the terminal voltages of the two dry cells, a constant voltage of 3 V. The problem is to find the seven unknown variables. From algebra, you know that to find n unknown quantities you must solve n simultaneous independent equations. From our discussion of Ohm's law in Section 2.2, you know that three of the necessary equations are

$$v_1 = i_1 R_1, \tag{2.13}$$

$$v_c = i_c R_c, \tag{2.14}$$

$$v_l = i_l R_l. \tag{2.15}$$

What about the other four equations?

The interconnection of circuit elements imposes constraints on the relationship between the terminal voltages and currents. These constraints are referred to as Kirchhoff's laws, after Gustav Kirchhoff, after Gustav Kirchhoff, who first stated them in a paper published in 1848. The two laws that state the constraints in mathematical form are known as Kirchhoff's current law and Kirchhoff's voltage law.

We can now state **Kirchhoff's current law**:

Kirchhoff's current law (KCL) ▶

> The algebraic sum of all the currents at any node in a circuit equals zero.

To use Kirchhoff's current law, an algebraic sign corresponding to a reference direction must be assigned to every current at the node. Assigning a positive sign to a current leaving a node requires assigning a negative sign to a current entering a node. Conversely, giving a negative sign to a current leaving a node requires giving a positive sign to a current entering a node.

Applying Kirchhoff's current law to the four nodes in the circuit shown in Fig. 2.15, using the convention that currents leaving a node are considered positive, yields four equations:

$$\text{node a} \qquad i_s - i_1 = 0, \tag{2.16}$$

$$\text{node b} \qquad i_1 + i_c = 0, \tag{2.17}$$

$$\text{node c} \qquad -i_c - i_l = 0, \tag{2.18}$$

$$\text{node d} \qquad i_l - i_s = 0. \tag{2.19}$$

Note that Eqs. 2.16–2.19 are not an independent set, because any one of the four can be derived from the other three. In any circuit with n nodes, $n - 1$ independent current equations can be derived from Kirchhoff's current law.[1] Let's disregard Eq. 2.19 so that we have six independent equations, namely, Eqs. 2.13–2.18. We need one more, which we can derive from Kirchhoff's voltage law.

Before we can state Kirchhoff's voltage law, we must define a **closed path** or **loop**. Starting at an arbitrarily selected node, we trace a closed path in a circuit through selected basic circuit elements and return to the original node without passing through any intermediate node more than once. The circuit shown in Fig. 2.15 has only one closed path or loop. For example, choosing node a as the starting point and tracing the circuit clockwise, we form the closed path by moving through nodes d, c, b, and back to node a. We can now state **Kirchhoff's voltage law**:

The algebraic sum of all the voltages around any closed path in a circuit equals zero.

◀ **Kirchhoff's voltage law (KVL)**

To use Kirchhoff's voltage law, we must assign an algebraic sign (reference direction) to each voltage in the loop. As we trace a closed path, a voltage will appear either as a rise or a drop in the tracing direction. Assigning a positive sign to a voltage rise requires assigning a negative sign to a voltage drop. Conversely, giving a negative sign to a voltage rise requires giving a positive sign to a voltage drop.

We now apply Kirchhoff's voltage law to the circuit shown in Fig. 2.15. We elect to trace the closed path clockwise, assigning a positive algebraic sign to voltage drops. Starting at node d leads to the expression

$$v_l - v_c + v_1 - v_s = 0, \tag{2.20}$$

[1] We say more about this observation in Chapter 4.

which represents the seventh independent equation needed to find the seven unknown circuit variables mentioned earlier.

The thought of having to solve seven simultaneous equations to find the current delivered by a pair of dry cells to a flashlight lamp is not very appealing. Thus in the coming chapters we introduce you to analytical techniques that will enable you to solve a simple one-loop circuit by writing a single equation. However, before moving on to a discussion of these circuit techniques, we need to make several observations about the detailed analysis of the flashlight circuit. In general, these observations are true and therefore are important to the discussions in subsequent chapters. They also support the contention that the flashlight circuit can be solved by defining a single unknown.

First, note that if you know the current in a resistor, you also know the voltage across the resistor, because current and voltage are directly related through Ohm's law. Thus you can associate one unknown variable with each resistor, either the current or the voltage. Choose, say, the current as the unknown variable. Then, once you solve for the unknown current in the resistor, you can find the voltage across the resistor. In general, if you know the current in a passive element, you can find the voltage across it, greatly reducing the number of simultaneous equations to be solved. For example, in the flashlight circuit, we eliminate the voltages v_c, v_l, and v_1 as unknowns. Thus at the outset we reduce the analytical task to solving four simultaneous equations rather than seven.

The second general observation relates to the consequences of connecting only two elements to form a node. According to Kirchhoff's current law, when only two elements connect to a node, if you know the current in one of the elements, you also know it in the second element. In other words, you need define only one unknown current for the two elements. When just two elements connect at a single node, the elements are said to be **in series**. The importance of this second observation is obvious when you note that each node in the circuit shown in Fig. 2.15 involves only two elements. Thus you need to define only one unknown current. The reason is that Eqs. 2.16–2.18 lead directly to

$$i_s = i_1 = -i_c = i_l, \qquad (2.21)$$

which states that if you know any one of the element currents, you know them all. For example, choosing to use i_s as the unknown eliminates i_1, i_c, and i_l. The problem is reduced to determining one unknown, namely, i_s.

Examples 2.6 and 2.7 illustrate how to write circuit equations based on Kirchhoff's laws. Example 2.8 illustrates how to use Kirchhoff's laws and Ohm's law to find an unknown current. Example 2.9 expands on the technique presented in Example 2.5 for constructing a circuit model for a device whose terminal characteristics are known.

Example 2.6 Using Kirchhoff's Current Law

Sum the currents at each node in the circuit shown in Fig. 2.16. Note that there is no connection dot (•) in the center of the diagram, where the 4 Ω branch crosses the branch containing the ideal current source i_a.

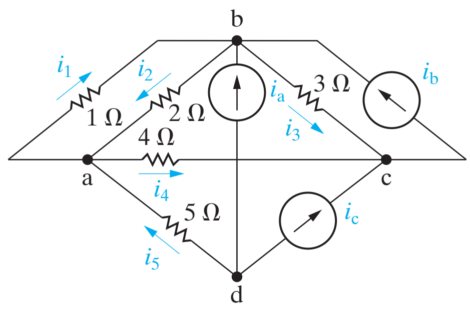


Figure 2.16 ▲ The circuit for Example 2.6.

Solution

In writing the equations, we use a positive sign for a current leaving a node. The four equations are

$$\text{node a} \quad i_1 + i_4 - i_2 - i_5 = 0,$$

$$\text{node b} \quad i_2 + i_3 - i_1 - i_b - i_a = 0,$$

$$\text{node c} \quad i_b - i_3 - i_4 - i_c = 0,$$

$$\text{node d} \quad i_5 + i_a + i_c = 0.$$

Example 2.7 Using Kirchhoff's Voltage Law

Sum the voltages around each designated path in the circuit shown in Fig. 2.17.

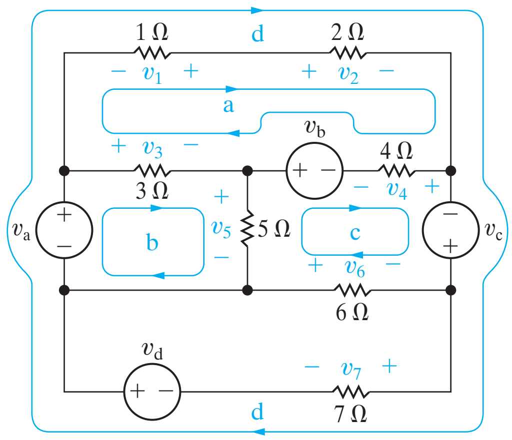


Figure 2.17 ▲ The circuit for Example 2.7.

Solution

In writing the equations, we use a positive sign for a voltage drop. The four equations are

$$\text{path a} \quad -v_1 + v_2 + v_4 - v_b - v_3 = 0,$$

$$\text{path b} \quad -v_a + v_3 + v_5 = 0,$$

$$\text{path c} \quad v_b - v_4 - v_c - v_6 - v_5 = 0,$$

$$\text{path d} \quad -v_a - v_1 + v_2 - v_c + v_7 - v_d = 0.$$

Example 2.8 Applying Ohm's Law and Kirchhoff's Laws to Find an Unknown Current

a) Use Kirchhoff's laws and Ohm's law to find i_o in the circuit shown in Fig. 2.18.

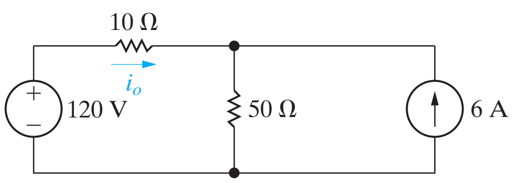


Figure 2.18 ▲ The circuit for Example 2.8.

b) Test the solution for i_o by verifying that the total power generated equals the total power dissipated.

Solution

a) We begin by redrawing the circuit and assigning an unknown current to the 50 Ω resistor and unknown voltages across the 10 Ω and 50 Ω resistors. Figure 2.19 shows the circuit. The nodes are labeled a, b, and c to aid the discussion.

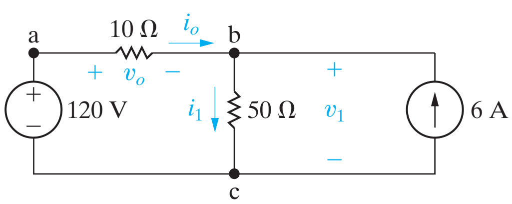


Figure 2.19 ▲ The circuit shown in Fig. 2.18, with the unknowns i_1, v_o, and v_1 defined.

Because i_o also is the current in the 120 V source, we have two unknown currents and therefore must derive two simultaneous equations involving i_o and i_1. We obtain one of the equations by applying Kirchhoff's current law to either node b or c. Summing the currents at node b and assigning a positive sign to the currents leaving the node gives

$$i_1 - i_o - 6 = 0.$$

We obtain the second equation from Kirchhoff's voltage law in combination with Ohm's law. Noting from Ohm's law that v_o is $10i_o$ and v_1 is $50i_1$, we sum the voltages around the closed path cabc to obtain

$$-120 + 10i_o + 50i_1 = 0.$$

In writing this equation, we assigned a positive sign to voltage drops in the clockwise direction. Solving these two equations for i_o and i_1 yields

$$i_o = -3 \text{ A} \quad \text{and} \quad i_1 = 3 \text{ A}.$$

b) The power dissipated in the 50 Ω resistor is

$$p_{50\Omega} = (3)^2(50) = 450 \text{ W}.$$

The power dissipated in the 10 Ω resistor is

$$p_{10\Omega} = (-3)^2(10) = 90 \text{ W}.$$

The power delivered to the 120 V source is

$$p_{120\text{V}} = -120i_o = -120(-3) = 360 \text{ W}.$$

The power delivered to the 6 A source is

$$p_{6\text{A}} = -v_1(6), \quad \text{but} \quad v_1 = 50i_1 = 150 \text{ V}.$$

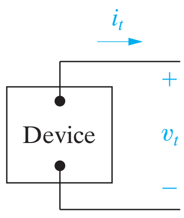
Therefore

$$p_{6\text{A}} = -150(6) = -900 \text{ W}.$$

The 6 A source is delivering 900 W, and the 120 V source is absorbing 360 W. The total power absorbed is 360 + 450 + 90 = 900 W. Therefore, the solution verifies that the power delivered equals the power absorbed.

Example 2.9 Constructing a Circuit Model Based on Terminal Measurements

The terminal voltage and terminal current were measured on the device shown in Fig. 2.20(a), and the values of v_t and i_t are tabulated in Fig. 2.20(b).



v_t (V)	i_t (A)
30	0
15	3
0	6

(b)

Figure 2.20 ▲ (a) Device and (b) data for Example 2.9.

a) Construct a circuit model of the device inside the box.

b) Using this circuit model, predict the power this device will deliver to a 10 Ω resistor.

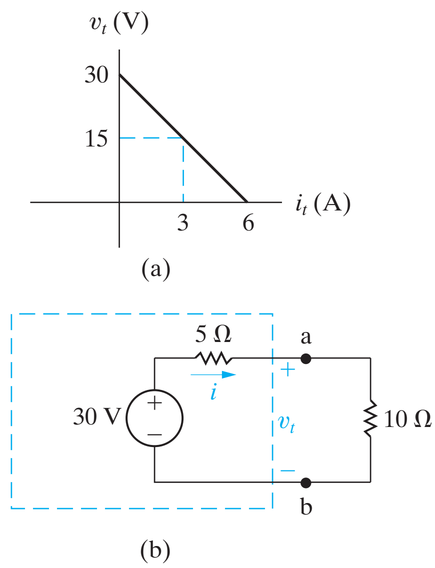


Figure 2.21 ▲ (a) The graph of v_t versus i_t for the device in Fig. 2.20(a). (b) The resulting circuit model for the device in Fig. 2.20(a), connected to a 10 Ω resistor.

Solution

a) Plotting the voltage as a function of the current yields the graph shown in Fig. 2.21(a). The equation of the line plotted is

$$v_t = 30 - 5i_t.$$

Now we need to identify the components of a circuit model that will produce the same relationship between voltage and current. Kirchhoff's voltage law tells us that the voltage drops across two components in series add. From the equation, one of those components produces a 30 V drop regardless of the current. This component can be modeled as an ideal independent voltage source. The other component produces a positive voltage drop in the direction of the current i_t. Because the voltage drop is proportional to the current, Ohm's law tells us that this component can be modeled as an ideal resistor with a value of 5 Ω. The resulting circuit model is depicted in the dashed box in Fig. 2.21(b).

b) Now we attach a 10 Ω resistor to the device in Fig. 2.21(b) to complete the circuit. Kirchhoff's current law tells us that the current in the 10 Ω resistor is the same as the current in the 5 Ω resistor. Using Kirchhoff's voltage law and Ohm's law, we can write the equation for the voltage drops around the circuit, starting at the voltage source and proceeding clockwise:

$$-30 + 5i + 10i = 0.$$

Solving for i, we get

$$i = 2\text{ A}.$$

Because this is the value of current flowing in the 10 Ω resistor, we can use the power equation $p = i^2R$ to compute the power delivered to this resistor:

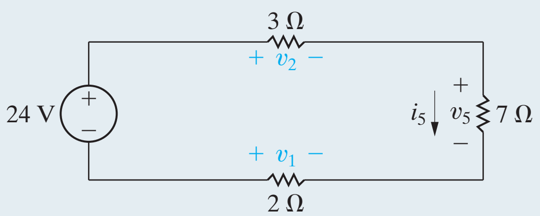
$$p_{10\Omega} = (2)^2(10) = 40\ \text{W}.$$

✓ ASSESSMENT PROBLEMS

Objective 2—Be able to state and use Ohm's law and Kirchhoff's current and voltage laws

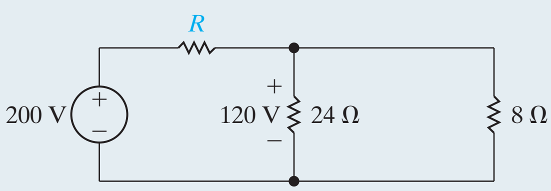
2.5 For the circuit shown, calculate (a) i_5; (b) v_1; (c) v_2; (d) v_5; and (e) the power delivered by the 24 V source.

Answer: (a) 2 A;
(b) –4 V;
(c) 6 V;
(d) 14 V;
(e) 48 W.



2.6 Use Ohm's law and Kirchhoff's laws to find the value of R in the circuit shown.

Answer: $R = 4\ \Omega$.

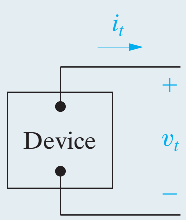


NOTE: Also try Chapter Problems 2.14, 2.17, 2.18, and 2.19.

2.7 a) The terminal voltage and terminal current were measured on the device shown. The values of v_t and i_t are provided in the table. Using these values, create the straight line plot of v_t versus i_t. Compute the equation of the line and use the equation to construct a circuit model for the device using an ideal voltage source and a resistor.

b) Use the model constructed in (a) to predict the power that the device will deliver to a 25 Ω resistor.

Answer: (a) A 25 V source in series with a 100 Ω resistor;
(b) 1 W.



(a)

v_t (V)	i_t (A)
25	0
15	0.1
5	0.2
0	0.25

(b)

2.8 Repeat Assessment Problem 2.7 but use the equation of the graphed line to construct a circuit model containing an ideal current source and a resistor.

Answer: (a) A 0.25 A current source connected between the terminals of a 100 Ω resistor;
(b) 1 W.

2.5 Analysis of a Circuit Containing Dependent Sources

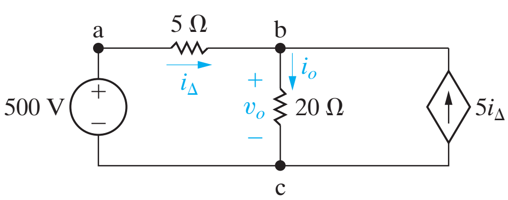


Figure 2.22 ▲ A circuit with a dependent source.

We conclude this introduction to elementary circuit analysis with a discussion of a circuit that contains a dependent source, as depicted in Fig. 2.22.

We want to use Kirchhoff's laws and Ohm's law to find v_o in this circuit. Before writing equations, it is good practice to examine the circuit diagram closely. This will help us identify the information that is known and the information we must calculate. It may also help us devise a strategy for solving the circuit using only a few calculations.

A look at the circuit in Fig. 2.22 reveals that

- Once we know i_o, we can calculate v_o using Ohm's law.
- Once we know i_Δ, we also know the current supplied by the dependent source $5i_\Delta$.
- The current in the 500 V source is i_Δ.

There are thus two unknown currents, i_Δ and i_o. We need to construct and solve two independent equations involving these two currents to produce a value for v_o.

From the circuit, notice the closed path containing the voltage source, the 5 Ω resistor, and the 20 Ω resistor. We can apply Kirchhoff's voltage law around this closed path. The resulting equation contains the two unknown currents:

$$500 = 5i_\Delta + 20i_o. \tag{2.22}$$

Now we need to generate a second equation containing these two currents. Consider the closed path formed by the 20 Ω resistor and the dependent current source. If we attempt to apply Kirchhoff's voltage law to this loop, we fail to develop a useful equation, because we don't know the value of the voltage across the dependent current source. In fact, the voltage across the dependent source is v_o, which is the voltage we are trying to compute. Writing an equation for this loop does not advance us toward a solution. For this same reason, we do not use the closed path containing the voltage source, the 5 Ω resistor, and the dependent source.

There are three nodes in the circuit, so we turn to Kirchhoff's current law to generate the second equation. Node a connects the voltage source and the 5 Ω resistor; as we have already observed, the current in these two elements is the same. Either node b or node c can be used to construct the second equation from Kirchhoff's current law. We select node b and produce the following equation:

$$i_o = i_\Delta + 5i_\Delta = 6i_\Delta. \tag{2.23}$$

Solving Eqs. 2.22 and 2.23 for the currents, we get

$$i_\Delta = 4 \text{ A},$$

$$i_o = 24 \text{ A}. \tag{2.24}$$

Using Eq. 2.24 and Ohm's law for the 20 Ω resistor, we can solve for the voltage v_o:

$$v_o = 20i_o = 480 \text{ V}.$$

Think about a circuit analysis strategy before beginning to write equations. As we have demonstrated, not every closed path provides an opportunity to write a useful equation based on Kirchhoff's voltage law. Not every node provides for a useful application of Kirchhoff's current law. Some preliminary thinking about the problem can help in selecting the most fruitful approach and the most useful analysis tools for a particular problem. Choosing a good approach and the appropriate tools will usually reduce the number and complexity of equations to be solved. Example 2.10 illustrates another application of Ohm's law and Kirchhoff's laws to a circuit with a dependent source. Example 2.11 involves a much more complicated circuit, but with a careful choice of analysis tools, the analysis is relatively uncomplicated.

Example 2.10 Applying Ohm's Law and Kirchhoff's Laws to Find an Unknown Voltage

a) Use Kirchhoff's laws and Ohm's law to find the voltage v_o as shown in Fig. 2.23.

b) Show that your solution is consistent with the constraint that the total power developed in the circuit equals the total power dissipated.

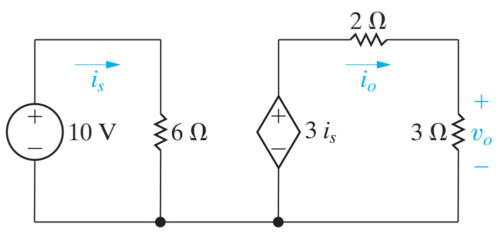


Figure 2.23 ▲ The circuit for Example 2.10.

Solution

a) A close look at the circuit in Fig. 2.23 reveals that:
- There are two closed paths, the one on the left with the current i_s and the one on the right with the current i_o.
- Once i_o is known, we can compute v_o.

We need two equations for the two currents. Because there are two closed paths and both have voltage sources, we can apply Kirchhoff's voltage law to each to give the following equations:

$$10 = 6i_s,$$

$$3i_s = 2i_o + 3i_o.$$

Solving for the currents yields

$$i_s = 1.67 \text{ A},$$

$$i_o = 1 \text{ A}.$$

Applying Ohm's law to the 3 Ω resistor gives the desired voltage:

$$v_o = 3i_o = 3 \text{ V}.$$

b) To compute the power delivered to the voltage sources, we use the power equation in the form $p = vi$. The power delivered to the independent voltage source is

$$p = (10)(-1.67) = -16.7 \text{ W}.$$

The power delivered to the dependent voltage source is

$$p = (3i_s)(-i_o) = (5)(-1) = -5 \text{ W}.$$

Both sources are developing power, and the total developed power is 21.7 W.

To compute the power delivered to the resistors, we use the power equation in the form $p = i^2R$. The power delivered to the 6 Ω resistor is

$$p = (1.67)^2(6) = 16.7 \text{ W}.$$

The power delivered to the 2 Ω resistor is

$$p = (1)^2(2) = 2 \text{ W}.$$

The power delivered to the 3 Ω resistor is

$$p = (1)^2(3) = 3 \text{ W}.$$

The resistors all dissipate power, and the total power dissipated is 21.7 W, equal to the total power developed in the sources.

Example 2.11 Applying Ohm's Law and Kirchhoff's Law in an Amplifier Circuit

The circuit in Fig. 2.24 represents a common configuration encountered in the analysis and design of transistor amplifiers. Assume that the values of all the circuit elements—R_1, R_2, R_C, R_E, V_{CC}, and V_0—are known.

a) Develop the equations needed to determine the current in each element of this circuit.

b) From these equations, devise a formula for computing i_B in terms of the circuit element values.

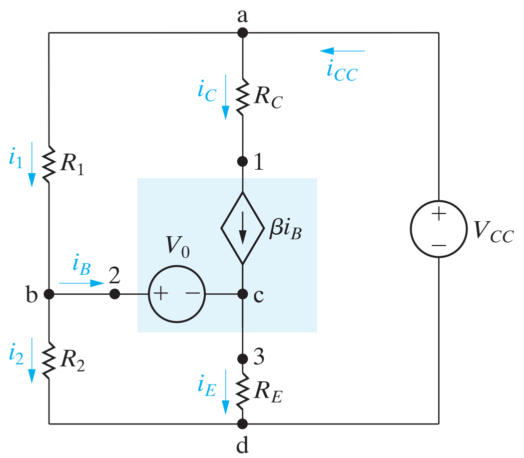


Figure 2.24 ▲ The circuit for Example 2.11.

Solution

A careful examination of the circuit reveals a total of six unknown currents, designated i_1, i_2, i_B, i_C, i_E, and i_{CC}. In defining these six unknown currents, we used the observation that the resistor R_C is in series with the dependent current source βi_B. We now must derive six independent equations involving these six unknowns.

a) We can derive three equations by applying Kirchhoff's current law to any three of the nodes a, b, c, and d. Let's use nodes a, b, and c and label the currents away from the nodes as positive:

$$(1) \quad i_1 + i_C - i_{CC} = 0,$$

$$(2) \quad i_B + i_2 - i_1 = 0,$$

$$(3) \quad i_E - i_B - i_C = 0.$$

A fourth equation results from imposing the constraint presented by the series connection of R_C and the dependent source:

$$(4) \quad i_C = \beta i_B.$$

We turn to Kirchhoff's voltage law in deriving the remaining two equations. We need to select two closed paths in order to use Kirchhoff's voltage law. Note that the voltage across the dependent current source is unknown, and that it cannot be determined from the source current βi_B. Therefore, we must select two closed paths that do not contain this dependent current source.

We choose the paths bcdb and badb and specify voltage drops as positive to yield

$$(5) \quad V_0 + i_E R_E - i_2 R_2 = 0,$$

$$(6) \quad -i_1 R_1 + V_{CC} - i_2 R_2 = 0.$$

b) To get a single equation for i_B in terms of the known circuit variables, you can follow these steps:

- Solve Eq. (6) for i_1, and substitute this solution for i_1 into Eq. (2).
- Solve the transformed Eq. (2) for i_2, and substitute this solution for i_2 into Eq. (5).
- Solve the transformed Eq. (5) for i_E, and substitute this solution for i_E into Eq. (3). Use Eq. (4) to eliminate i_C in Eq. (3).
- Solve the transformed Eq. (3) for i_B, and rearrange the terms to yield

$$i_B = \frac{(V_{CC}R_2)/(R_1 + R_2) - V_0}{(R_1R_2)/(R_1 + R_2) + (1 + \beta)R_E}. \quad (2.25)$$

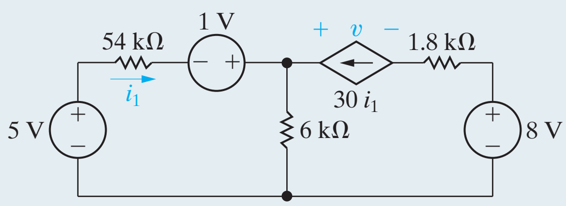
Problem 2.27 asks you to verify these steps. Note that once we know i_B, we can easily obtain the remaining currents.

✓ASSESSMENT PROBLEMS

Objective 3—Know how to calculate power for each element in a simple circuit

2.9 For the circuit shown find (a) the current i_1 in microamperes, (b) the voltage v in volts, (c) the total power generated, and (d) the total power absorbed.

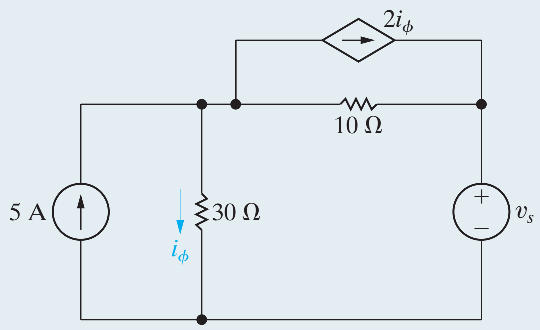
Answer: (a) 25 μA;
(b) –2 V;
(c) 6150 μW;
(d) 6150 μW.



2.10 The current i_ϕ in the circuit shown is 2 A. Calculate

a) v_s,
b) the power absorbed by the independent voltage source,
c) the power delivered by the independent current source,
d) the power delivered by the controlled current source,
e) the total power dissipated in the two resistors.

Answer: (a) 70 V;
(b) 210 W;
(c) 300 W;
(d) 40 W;
(e) 130 W.



NOTE: Also try Chapter Problems 2.24 and 2.28.

Practical Perspective

Electrical Safety

At the beginning of this chapter, we said that current through the body can cause injury. Let's examine this aspect of electrical safety.

You might think that electrical injury is due to burns. However, that is not the case. The most common electrical injury is to the nervous system. Nerves use electrochemical signals, and electric currents can disrupt those signals. When the current path includes only skeletal muscles, the effects can include temporary paralysis (cessation of nervous signals) or involuntary muscle contractions, which are generally not life threatening. However, when the current path includes nerves and muscles that control the supply of oxygen to the brain, the problem is much more serious. Temporary paralysis of these muscles can stop a person from breathing, and a sudden muscle contraction can disrupt the signals that regulate heartbeat. The result is a halt in the flow of oxygenated blood to the brain, causing death in a few minutes unless emergency aid is given immediately. Table 2.1 shows a range of physiological reactions to various current levels. The numbers in this table are approximate; they are obtained from an analysis of accidents because, obviously, it is not ethical to perform electrical experiments on people. Good electrical design will limit current to a few milliamperes or less under all possible conditions.

TABLE 2.1 Physiological Reactions to Current Levels in Humans

Physiological Reaction	Current
Barely perceptible	3–5 mA
Extreme pain	35–50 mA
Muscle paralysis	50–70 mA
Heart stoppage	500 mA

Note: Data taken from W. F. Cooper, *Electrical Safety Engineering*, 2d ed. (London: Butterworth, 1986); and C. D. Winburn, *Practical Electrical Safety* (Monticello, N.Y.: Marcel Dekker, 1988).

Now we develop a simplified electrical model of the human body. The body acts as a conductor of current, so a reasonable starting point is to model the body using resistors. Figure 2.25 shows a potentially dangerous situation. A voltage difference exists between one arm and one leg of a human being. Figure 2.25(b) shows an electrical model of the human body in Fig. 2.25(a). The arms, legs, neck, and trunk (chest and abdomen) each have a characteristic resistance. Note that the path of the current is through the trunk, which contains the heart, a potentially deadly arrangement.

NOTE: Assess your understanding of the Practical Perspective by solving Chapter Problems 2.34–2.38.

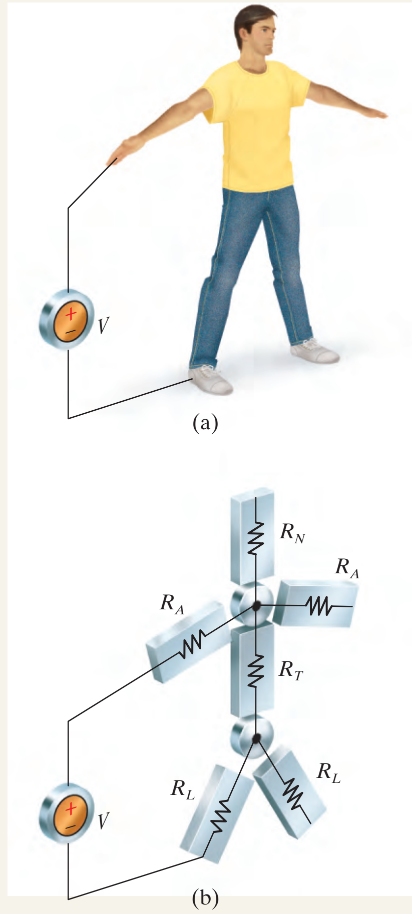


Figure 2.25 ▲ (a) A human body with a voltage difference between one arm and one leg. (b) A simplified model of the human body with a voltage difference between one arm and one leg.

Summary

- The circuit elements introduced in this chapter are voltage sources, current sources, and resistors:
  - An **ideal voltage source** maintains a prescribed voltage regardless of the current in the device. An **ideal current source** maintains a prescribed current regardless of the voltage across the device. Voltage and current sources are either **independent**, that is, not influenced by any other current or voltage in the circuit; or **dependent**, that is, determined by some other current or voltage in the circuit. (See pages 24 and 25.)
  - A **resistor** constrains its voltage and current to be proportional to each other. The value of the proportional constant relating voltage and current in a resistor is called its **resistance** and is measured in ohms. (See page 28.)
- **Ohm's law** establishes the proportionality of voltage and current in a resistor. Specifically,

$$v = iR \tag{2.26}$$

if the current flow in the resistor is in the direction of the voltage drop across it, or

$$v = -iR \qquad (2.27)$$

if the current flow in the resistor is in the direction of the voltage rise across it. (See page 29.)

- By combining the equation for power, $p = vi$, with Ohm's law, we can determine the power absorbed by a resistor:

$$p = i^2R = v^2/R. \qquad (2.28)$$

(See page 30.)

- Circuits are described by nodes and closed paths. A **node** is a point where two or more circuit elements join. When just two elements connect to form a node, they are said to be **in series**. A **closed path** is a loop traced through connecting elements, starting and ending at the same node and encountering intermediate nodes only once each. (See pages 36–38.)

- The voltages and currents of interconnected circuit elements obey Kirchhoff's laws:
  - **Kirchhoff's current law** states that the algebraic sum of all the currents at any node in a circuit equals zero. (See page 36.)
  - **Kirchhoff's voltage law** states that the algebraic sum of all the voltages around any closed path in a circuit equals zero. (See page 37.)

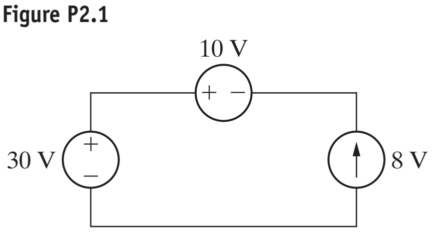
- A circuit is solved when the voltage across and the current in every element have been determined. By combining an understanding of independent and dependent sources, Ohm's law, and Kirchhoff's laws, we can solve many simple circuits.

Problems

Section 2.1

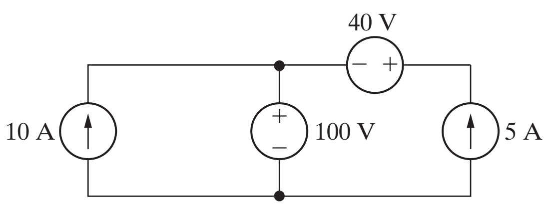
2.1 a) Is the interconnection of ideal sources in the circuit in Fig. P2.1 valid? Explain.
b) Identify which sources are developing power and which sources are absorbing power.
c) Verify that the total power developed in the circuit equals the total power absorbed.
d) Repeat (a)–(c), reversing the polarity of the 10 V source.

Figure P2.1



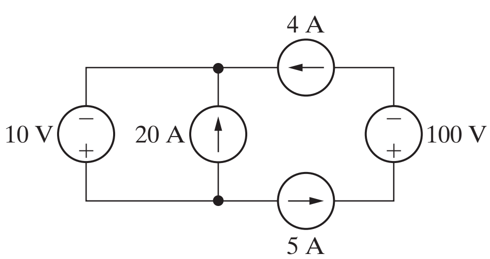
2.2 If the interconnection in Fig. P2.2 is valid, find the power developed by the current sources. If the interconnection is not valid, explain why.

Figure P2.2



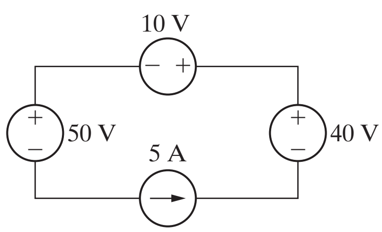
2.3 If the interconnection in Fig. P2.3 is valid, find the total power developed by the voltage sources. If the interconnection is not valid, explain why.

Figure P2.3



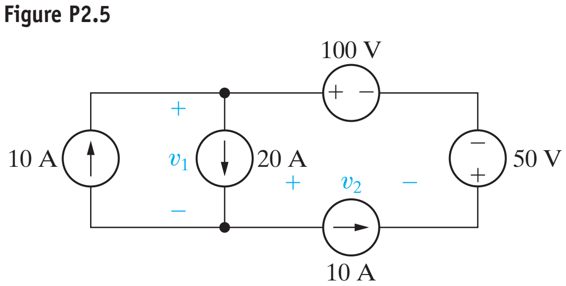
2.4 If the interconnection in Fig. P2.4 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.4



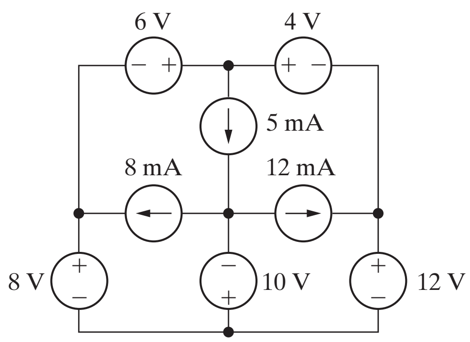
2.5 The interconnection of ideal sources can lead to an indeterminate solution. With this thought in mind, explain why the solutions for v_1 and v_2 in the circuit in Fig. P2.5 are not unique.

Figure P2.5



2.6 If the interconnection in Fig. P2.6 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

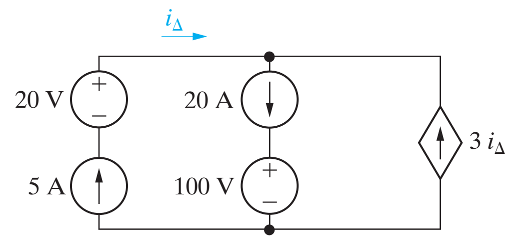
Figure P2.6



2.7 a) Is the interconnection in Fig. P2.7 valid? Explain.

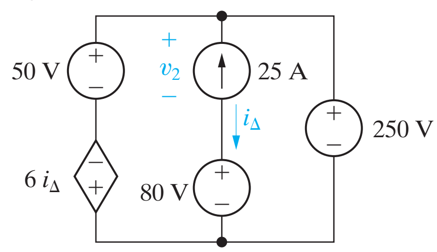
b) Can you find the total energy developed in the circuit? Explain.

Figure P2.7



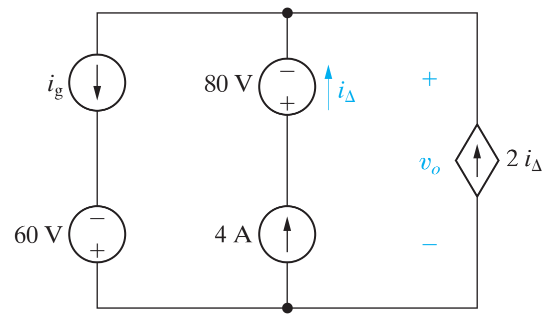
2.8 If the interconnection in Fig. P2.8 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.8



2.9 Find the total power developed in the circuit in Fig. P2.9 if $v_o = 100$ V and $i_g = 12$ A.

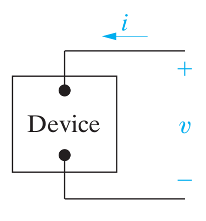
Figure P2.9



Sections 2.2–2.3

2.10 The terminal voltage and terminal current were measured on the device shown in Fig. P2.10(a). The values of v and i are given in the table of Fig. P2.10(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor.

Figure P2.10



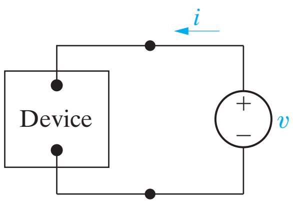
(a)

i (mA)	v (V)
−20	−160
−10	−80
10	80
20	160
30	240

(b)

2.11 A variety of voltage source values were applied to the device shown in Fig. P2.11(a). The power absorbed by the device for each value of voltage is recorded in the table given in Fig. P2.11(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor.

Figure P2.11



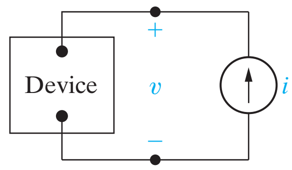
(a)

v (V)	p (mW)
−10	25.0
−5	6.25
5	6.25
10	25.0
15	56.25
20	100

(b)

2.12 A variety of current source values were applied to the device shown in Fig. P2.12(a). The power absorbed by the device for each value of current is recorded in the table given in Fig. P2.12(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor.

Figure P2.12



(a)

i (A)	p (W)
2	100
4	400
6	900
8	1600
10	2500
12	3600

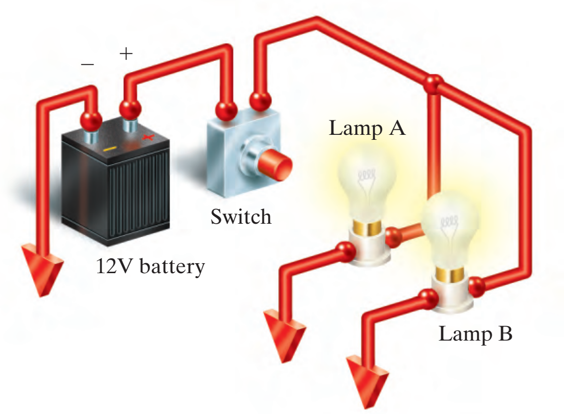
(b)

2.13 A pair of automotive headlamps is connected to a 12 V battery via the arrangements shown in Fig. P2.13. In the figure, the triangular symbol ▼ is used to indicate that the terminal is connected directly to the metal frame of the car.

a) Construct a circuit model using resistors and an independent voltage source.

b) Identify the correspondence between the ideal circuit element and the symbol component that it represents.

Figure P2.13

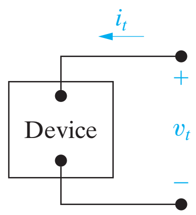


2.14 The voltage and current were measured at the terminals of the device shown in Fig. P2.14(a). The results are tabulated in Fig. P2.14(b).

a) Construct a circuit model for this device using an ideal current source and a resistor.

b) Use the model to predict the amount of power the device will deliver to a 5 Ω resistor.

Figure P2.14



(a)

v_t (V)	i_t (A)
100	0
180	4
260	8
340	12
420	16

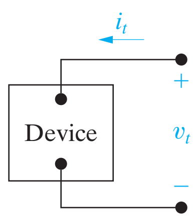
(b)

2.15 The voltage and current were measured at the terminals of the device shown in Fig. P2.15(a). The results are tabulated in Fig. P2.15(b).

a) Construct a circuit model for this device using an ideal voltage source and a resistor.

b) Use the model to predict the value of i_t when v_t is zero.

Figure P2.15



(a)

v_t (V)	i_t (A)
50	0
58	2
66	4
74	6
82	8
90	10

(b)

2.16 The table in Fig. P2.16(a) gives the relationship between the terminal current and voltage of the practical constant current source shown in Fig. P2.16(b).

a) Plot i_s versus v_s.

b) Construct a circuit model of this current source that is valid for $0 \leq v_s \leq 30$ V, based on the equation of the line plotted in (a).

c) Use your circuit model to predict the current delivered to a 3 kΩ resistor.

d) Use your circuit model to predict the open-circuit voltage of the current source.

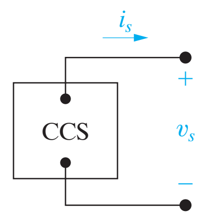
e) What is the actual open-circuit voltage?

f) Explain why the answers to (d) and (e) are not the same.

Figure P2.16

i_s (mA)	v_s (V)
40	0
35	10
30	20
25	30
18	40
8	50
0	55

(a)



(b)

2.17 The table in Fig. P2.17(a) gives the relationship between the terminal voltage and current of the practical constant voltage source shown in Fig. P2.17(b).

a) Plot v_s versus i_s.

b) Construct a circuit model of the practical source that is valid for $0 \leq i_s \leq 225$ mA, based on the equation of the line plotted in (a). (Use an ideal voltage source in series with an ideal resistor.)

c) Use your circuit model to predict the current delivered to a 400 Ω resistor connected to the terminals of the practical source.

d) Use your circuit model to predict the current delivered to a short circuit connected to the terminals of the practical source.

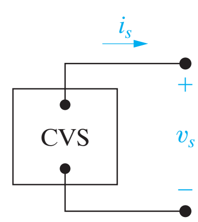
e) What is the actual short-circuit current?

f) Explain why the answers to (d) and (e) are not the same.

Figure P2.17

v_s (V)	i_s (mA)
75	0
60	75
45	150
30	225
20	300
10	400
0	500

(a)

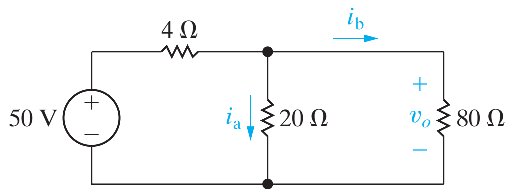


(b)

Section 2.4

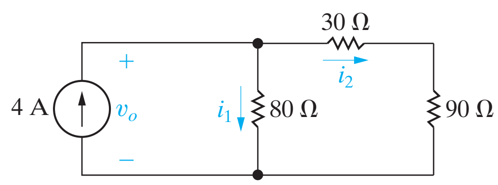
2.18 Given the circuit shown in Fig. P2.18, find
PSPICE
a) the value of i_a,
b) the value of i_b,
c) the value of v_o,
d) the power dissipated in each resistor,
e) the power delivered by the 50 V source.

Figure P2.18



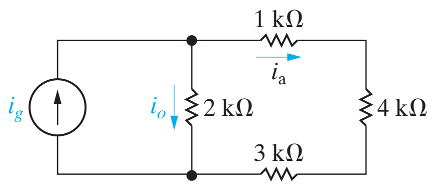
2.19 a) Find the currents i_1 and i_2 in the circuit in Fig. P2.19.
PSPICE
b) Find the voltage v_o.
c) Verify that the total power developed equals the total power dissipated.

Figure P2.19



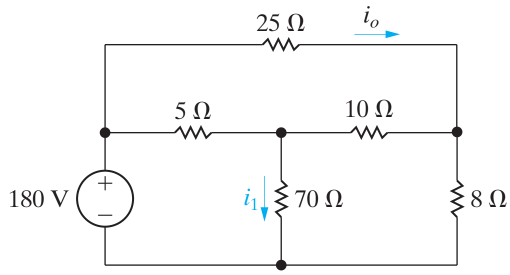
2.20 The current i_a in the circuit shown in Fig. P2.20 is 2 mA. Find (a) i_o; (b) i_g; and (c) the power delivered by the independent current source.
PSPICE

Figure P2.20



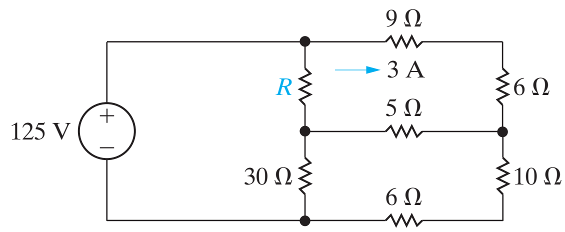
2.21 The current i_o in the circuit in Fig. P2.21 is 4 A.
PSPICE
a) Find i_1.
b) Find the power dissipated in each resistor.
c) Verify that the total power dissipated in the circuit equals the power developed by the 180 V source.

Figure P2.21



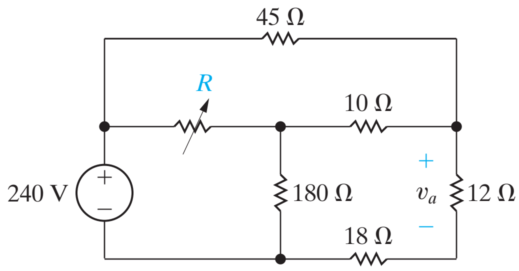
2.22 For the circuit shown in Fig. P2.22, find (a) R and (b) the power supplied by the 125 V source.
PSPICE

Figure P2.22



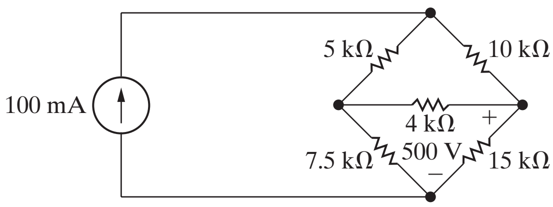
2.23 The variable resistor R in the circuit in Fig. P2.23 is adjusted until v_a equals 60 V. Find the value of R.
PSPICE

Figure P2.23



2.24 The voltage across the 15 kΩ resistor in the circuit in Fig. P2.24 is 500 V, positive at the upper terminal.
PSPICE
a) Find the power dissipated in each resistor.
b) Find the power supplied by the 100 mA ideal current source.
c) Verify that the power supplied equals the total power dissipated.

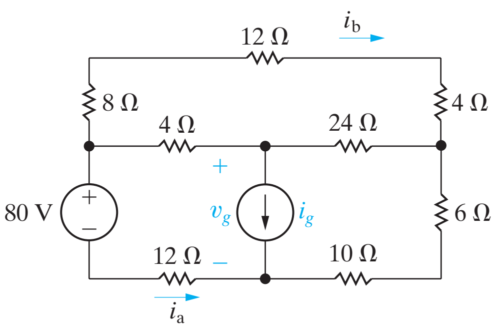
Figure P2.24



2.25 The currents i_a and i_b in the circuit in Fig. P2.25 are 4 A and 2 A, respectively.
PSPICE

a) Find i_g.
b) Find the power dissipated in each resistor.
c) Find v_g.
d) Show that the power delivered by the current source is equal to the power absorbed by all the other elements.

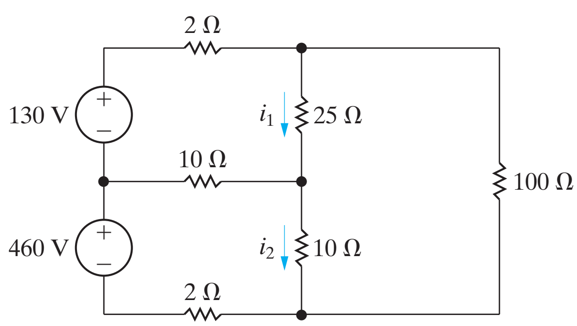
Figure P2.25



2.26 The currents i_1 and i_2 in the circuit in Fig. P2.26 are 10 A and 25 A, respectively.

a) Find the power supplied by each voltage source.
b) Show that the total power supplied equals the total power dissipated in the resistors.

Figure P2.26

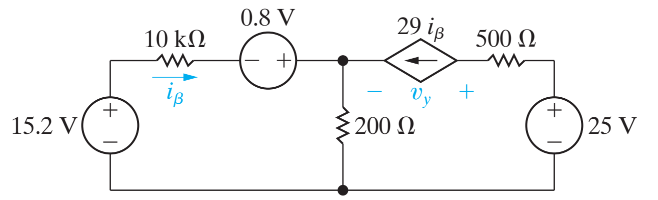


Section 2.5

2.27 Derive Eq. 2.25. *Hint:* Use Eqs. (3) and (4) from Example 2.11 to express i_E as a function of i_B. Solve Eq. (2) for i_2 and substitute the result into both Eqs. (5) and (6). Solve the "new" Eq. (6) for i_1 and substitute this result into the "new" Eq. (5). Replace i_E in the "new" Eq. (5) and solve for i_B. Note that because i_{CC} appears only in Eq. (1), the solution for i_B involves the manipulation of only five equations.

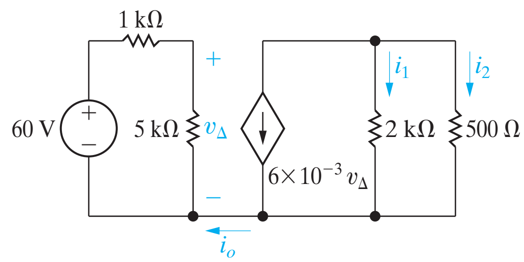
2.28 a) Find the voltage v_y in the circuit in Fig. P2.28.
PSPICE
b) Show that the total power generated in the circuit equals the total power absorbed.

Figure P2.28



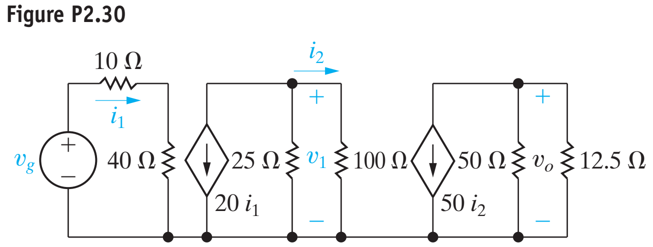
2.29 Find (a) i_o, (b) i_1, and (c) i_2 in the circuit in Fig. P2.29.
PSPICE

Figure P2.29



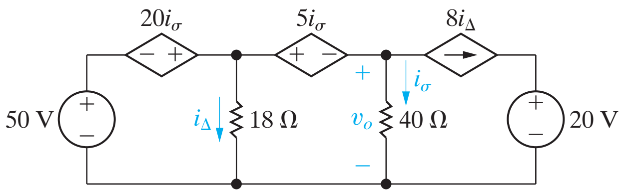
2.30 Find v_1 and v_g in the circuit shown in Fig. P2.30 when v_0 equals 250 mV. (*Hint:* Start at the right end of the circuit and work back toward v_g.)
PSPICE

Figure P2.30



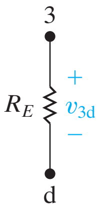
2.31 PSPICE For the circuit shown in Fig. P2.31, calculate (a) i_Δ and v_o and (b) show that the power developed equals the power absorbed.

Figure P2.31



2.32 PSPICE For the circuit shown in Fig. 2.24, $R_1 = 20\,\text{k}\Omega$, $R_2 = 80\,\text{k}\Omega$, $R_C = 500\,\Omega$, $R_E = 100\,\Omega$, $V_{CC} = 15$ V, $V_0 = 200$ mV, and $\beta = 39$. Calculate i_B, i_C, i_E, v_{3d}, v_{bd}, i_2, i_1, v_{ab}, i_{CC}, and v_{13}. (*Note:* In the double subscript notation on voltage variables, the first subscript is positive with respect to the second subscript. See Fig. P2.32.)

Figure P2.32



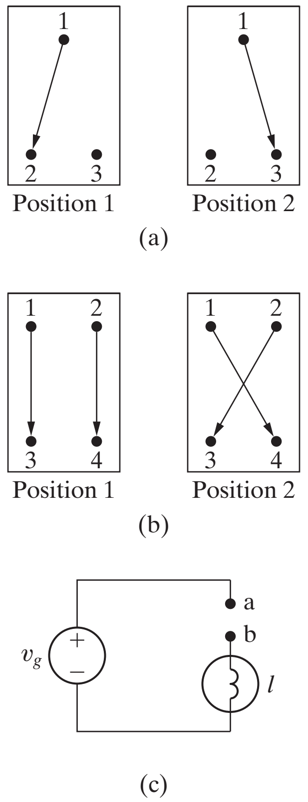
Sections 2.1–2.5

2.33 DESIGN PROBLEM It is often desirable in designing an electric wiring system to be able to control a single appliance from two or more locations, for example, to control a lighting fixture from both the top and bottom of a stairwell. In home wiring systems, this type of control is implemented with three-way and four-way switches. A three-way switch is a three-terminal, two-position switch, and a four-way switch is a four-terminal, two-position switch. The switches are shown schematically in Fig. P2.33(a), which illustrates a three-way switch, and P2.33(b), which illustrates a four-way switch.

a) Show how two three-way switches can be connected between a and b in the circuit in Fig. P2.33(c) so that the lamp l can be turned ON or OFF from two locations.

b) If the lamp (appliance) is to be controlled from more than two locations, four-way switches are used in conjunction with two three-way switches. One four-way switch is required for each location in excess of two. Show how one four-way switch plus two three-way switches can be connected between a and b in Fig. P2.33(c) to control the lamp from three locations. (*Hint:* The four-way switch is placed between the three-way switches.)

Figure P2.33



2.34 PRACTICAL PERSPECTIVE Suppose the power company installs some equipment that could provide a 250 V shock to a human being. Is the current that results dangerous enough to warrant posting a warning sign and taking other precautions to prevent such a shock? Assume that if the source is 250 V, the resistance of the arm is 400 Ω, the resistance of the trunk is 50 Ω, and the resistance of the leg is 200 Ω. Use the model given in Fig. 2.25(b).

2.35 PRACTICAL PERSPECTIVE Based on the model and circuit shown in Fig. 2.25, draw a circuit model of the path of current through the human body for a person touching a voltage source with both hands who has both feet at the same potential as the negative terminal of the voltage source.

2.36 PRACTICAL PERSPECTIVE a) Using the values of resistance for arm, leg, and trunk provided in Problem 2.34, calculate the power dissipated in the arm, leg, and trunk.

b) The specific heat of water is $4.18 \times 10_3$ J/kg°C, so a mass of water M (in kilograms) heated by a power P (in watts) undergoes a rise in temperature at a rate given by

$$\frac{dT}{dt} = \frac{2.39 \times 10^{-4} P}{M} \text{°C/s}.$$

Assuming that the mass of an arm is 4 kg, the mass of a leg is 10 kg, and the mass of a trunk is 25 kg, and that the human body is mostly water, how many seconds does it take the arm, leg, and trunk to rise the 5°C that endangers living tissue?

c) How do the values you computed in (b) compare with the few minutes it takes for oxygen starvation to injure the brain?

2.37 PRACTICAL PERSPECTIVE A person accidently grabs conductors connected to each end of a dc voltage source, one in each hand.

a) Using the resistance values for the human body provided in Problem 2.34, what is the minimum source voltage that can produce electrical shock sufficient to cause paralysis, preventing the person from letting go of the conductors?

b) Is there a significant risk of this type of accident occurring while servicing a personal computer, which typically has 5 V and 12 V sources?

2.38 PRACTICAL PERSPECTIVE To understand why the voltage level is not the sole determinant of potential injury due to electrical shock, consider the case of a static electricity shock mentioned in the Practical Perspective at the start of this chapter. When you shuffle your feet across a carpet, your body becomes charged. The effect of this charge is that your entire body represents a voltage potential. When you touch a metal doorknob, a voltage difference is created between you and the doorknob, and current flows—but the conduction material is air, not your body!

Suppose the model of the space between your hand and the doorknob is a 1 MΩ resistance. What voltage potential exists between your hand and the doorknob if the current causing the mild shock is 3 mA?