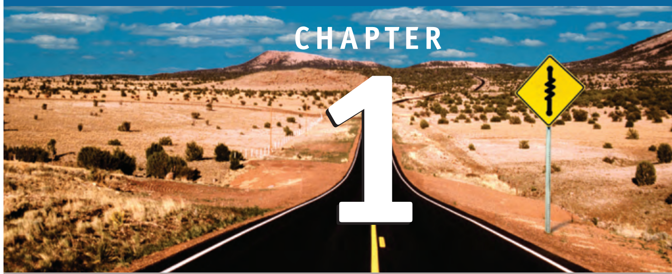

Circuit Variables

CHAPTER CONTENTS

✓ CHAPTER OBJECTIVES

1. Understand and be able to use SI units and the standard prefixes for powers of 10.
2. Know and be able to use the definitions of *voltage* and *current*.
3. Know and be able to use the definitions of *power* and *energy*.
4. Be able to use the passive sign convention to calculate the power for an ideal basic circuit element given its voltage and current.

Electrical engineering is an exciting and challenging profession for anyone who has a genuine interest in, and aptitude for, applied science and mathematics. Over the past century and a half, electrical engineers have played a dominant role in the development of systems that have changed the way people live and work. Satellite communication links, telephones, digital computers, televisions, diagnostic and surgical medical equipment, assembly-line robots, and electrical power tools are representative components of systems that define a modern technological society. As an electrical engineer, you can participate in this ongoing technological revolution by improving and refining these existing systems and by discovering and developing new systems to meet the needs of our ever-changing society.

As you embark on the study of circuit analysis, you need to gain a feel for where this study fits into the hierarchy of topics that comprise an introduction to electrical engineering. Hence we begin by presenting an overview of electrical engineering, some ideas about an engineering point of view as it relates to circuit analysis, and a review of the international system of units.

We then describe generally what circuit analysis entails. Next, we introduce the concepts of voltage and current. We follow these concepts with discussion of an ideal basic element and the need for a polarity reference system. We conclude the chapter by describing how current and voltage relate to power and energy.

1.1 Electrical Engineering: An Overview

Electrical engineering is the profession concerned with systems that produce, transmit, and measure electric signals. Electrical engineering combines the physicist's models of natural phenomena with the mathematician's tools for manipulating those models to produce systems that meet practical needs. Electrical systems pervade our lives; they are found in homes, schools, workplaces, and transportation vehicles everywhere. We begin by presenting a few examples from each of the five major classifications of electrical systems:

- communication systems
- computer systems
- control systems
- power systems
- signal-processing systems

Then we describe how electrical engineers analyze and design such systems.

Communication systems are electrical systems that generate, transmit, and distribute information. Well-known examples include television equipment, such as cameras, transmitters, receivers, and VCRs; radio telescopes, used to explore the universe; satellite systems, which return images of other planets and our own; radar systems, used to coordinate plane flights; and telephone systems.

Figure 1.1 depicts the major components of a modern telephone system. Starting at the left of the figure, inside a telephone, a microphone turns sound waves into electric signals. These signals are carried to a switching center where they are combined with the signals from tens, hundreds, or thousands of other telephones. The combined signals leave the switching center; their form depends on the distance they must travel. In our example, they are sent through wires in underground coaxial cables to a microwave transmission station. Here, the signals are transformed into microwave frequencies and broadcast from a transmission antenna through air and space, via a communications satellite, to a receiving antenna. The microwave receiving station translates the microwave signals into a form suitable for further transmission, perhaps as pulses of light to be sent through fiber-optic cable. On arrival at the second switching center, the combined signals are separated, and each is routed to the appropriate telephone, where an earphone acts as a speaker to convert the received electric signals back into sound waves. At each stage of the process, electric circuits operate on the signals. Imagine the challenge involved in designing, building, and operating each circuit in a way that guarantees that all of the hundreds of thousands of simultaneous calls have high-quality connections.

Computer systems use electric signals to process information ranging from word processing to mathematical computations. Systems range in size and power from pocket calculators to personal computers to supercomputers that perform such complex tasks as processing weather data and modeling chemical interactions of complex organic molecules. These systems include networks of microcircuits, or integrated circuits—postage-stampsized assemblies of hundreds, thousands, or millions of

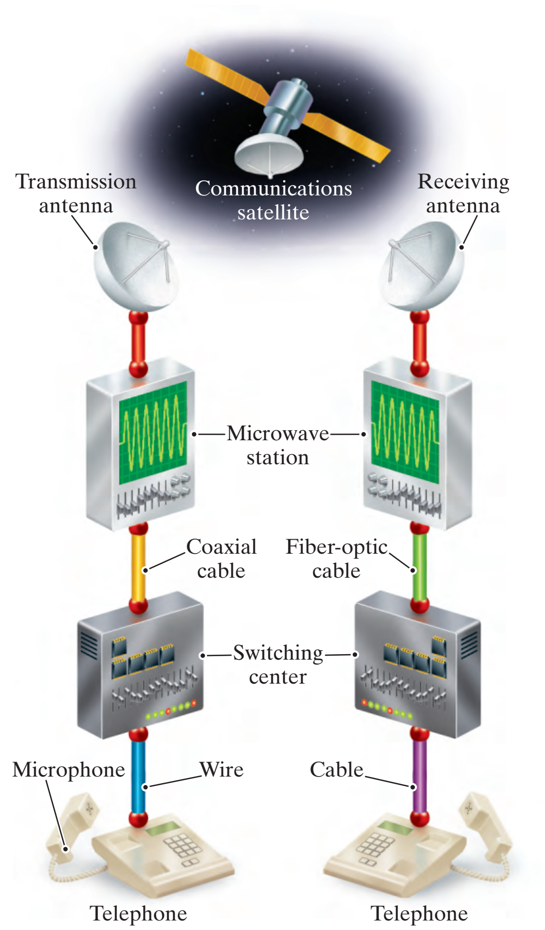

Figure 1.1 ▲ A telephone system.

electrical components that often operate at speeds and power levels close to fundamental physical limits, including the speed of light and the thermodynamic laws.

Control systems use electric signals to regulate processes. Examples include the control of temperatures, pressures, and flow rates in an oil refinery; the fuel-air mixture in a fuel-injected automobile engine; mechanisms such as the motors, doors, and lights in elevators; and the locks in the Panama Canal. The autopilot and autolanding systems that help to fly and land airplanes are also familiar control systems.

Power systems generate and distribute electric power. Electric power, which is the foundation of our technology-based society, usually is generated in large quantities by nuclear, hydroelectric, and thermal (coal-, oil-, or gas-fired) generators. Power is distributed by a grid of conductors that crisscross the country. A major challenge in designing and operating such a system is to provide sufficient redundancy and control so that failure of any piece of equipment does not leave a city, state, or region completely without power.

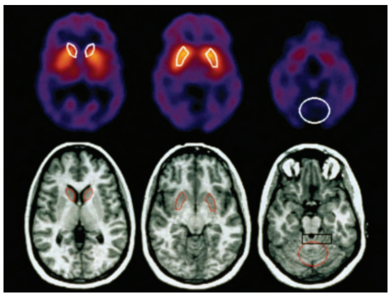

Figure 1.2 ▲ A CT scan of an adult head.

Signal-processing systems act on electric signals that represent information. They transform the signals and the information contained in them into a more suitable form. There are many different ways to process the signals and their information. For example, image-processing systems gather massive quantities of data from orbiting weather satellites, reduce the amount of data to a manageable level, and transform the remaining data into a video image for the evening news broadcast. A computerized tomography (CT) scan is another example of an image-processing system. It takes signals generated by a special X-ray machine and transforms them into an image such as the one in Fig. 1.2. Although the original X-ray signals are of little use to a physician, once they are processed into a recognizable image the information they contain can be used in the diagnosis of disease and injury.

Considerable interaction takes place among the engineering disciplines involved in designing and operating these five classes of systems. Thus communications engineers use digital computers to control the flow of information. Computers contain control systems, and control systems contain computers. Power systems require extensive communications systems to coordinate safely and reliably the operation of components, which may be spread across a continent. A signal-processing system may involve a communications link, a computer, and a control system.

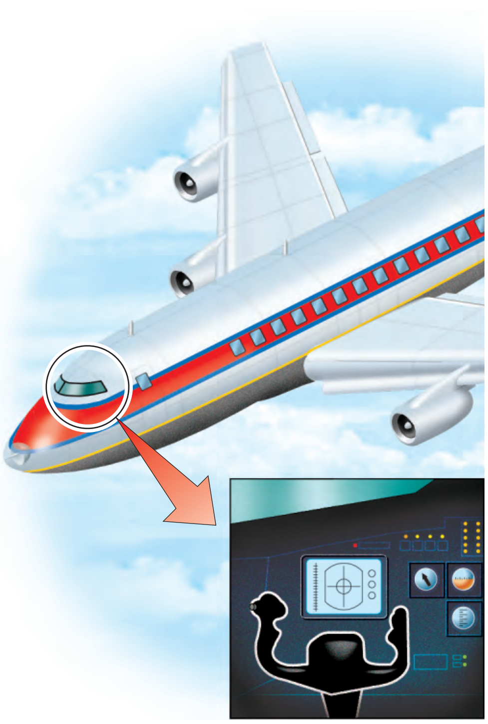

Figure 1.3 ▲ An airplane.

A good example of the interaction among systems is a commercial airplane, such as the one shown in Fig. 1.3. A sophisticated communications system enables the pilot and the air traffic controller to monitor the plane's location, permitting the air traffic controller to design a safe flight path for all of the nearby aircraft and enabling the pilot to keep the plane on its designated path. On the newest commercial airplanes, an onboard computer system is used for managing engine functions, implementing the navigation and flight control systems, and generating video information screens in the cockpit. A complex control system uses cockpit commands to adjust the position and speed of the airplane, producing the appropriate signals to the engines and the control surfaces (such as the wing flaps, ailerons, and rudder) to ensure the plane remains safely airborne and on the desired flight path. The plane must have its own power system to stay aloft and to provide and distribute the electric power needed to keep the cabin lights on, make the coffee, and show the movie. Signal-processing systems reduce the noise in air traffic communications and transform information about the plane's location into the more meaningful form of a video display in the cockpit. Engineering challenges abound in the design of each of these systems and their integration into a coherent whole. For example, these systems must operate in widely varying and unpredictable environmental conditions. Perhaps the most important engineering challenge is to guarantee that sufficient redundancy is incorporated in the designs to ensure that passengers arrive safely and on time at their desired destinations.

Although electrical engineers may be interested primarily in one area, they must also be knowledgeable in other areas that interact with this area of interest. This interaction is part of what makes electrical engineering a challenging and exciting profession. The emphasis in engineering is on making things work, so an engineer is free to acquire and use any technique, from any field, that helps to get the job done.

Circuit Theory

In a field as diverse as electrical engineering, you might well ask whether all of its branches have anything in common. The answer is yes—electric circuits. An **electric circuit** is a mathematical model that approximates the behavior of an actual electrical system. As such, it provides an important foundation for learning—in your later courses and as a practicing engineer—the details of how to design and operate systems such as those just described. The models, the mathematical techniques, and the language of circuit theory will form the intellectual framework for your future engineering endeavors.

Note that the term *electric circuit* is commonly used to refer to an actual electrical system as well as to the model that represents it. In this text, when we talk about an electric circuit, we always mean a model, unless otherwise stated. It is the modeling aspect of circuit theory that has broad applications across engineering disciplines.

Circuit theory is a special case of electromagnetic field theory: the study of static and moving electric charges. Although generalized field theory might seem to be an appropriate starting point for investigating electric signals, its application is not only cumbersome but also requires the use of advanced mathematics. Consequently, a course in electromagnetic field theory is not a prerequisite to understanding the material in this book. We do, however, assume that you have had an introductory physics course in which electrical and magnetic phenomena were discussed.

Three basic assumptions permit us to use circuit theory, rather than electromagnetic field theory, to study a physical system represented by an electric circuit. These assumptions are as follows:

1. *Electrical effects happen instantaneously throughout a system.* We can make this assumption because we know that electric signals travel at or near the speed of light. Thus, if the system is physically small, electric signals move through it so quickly that we can consider them to affect every point in the system simultaneously. A system that is small enough so that we can make this assumption is called a **lumped-parameter system**.

2. *The net charge on every component in the system is always zero.* Thus no component can collect a net excess of charge, although some components, as you will learn later, can hold equal but opposite separated charges.
3. *There is no magnetic coupling between the components in a system.* As we demonstrate later, magnetic coupling can occur *within* a component.

That's it; there are no other assumptions. Using circuit theory provides simple solutions (of sufficient accuracy) to problems that would become hopelessly complicated if we were to use electromagnetic field theory. These benefits are so great that engineers sometimes specifically design electrical systems to ensure that these assumptions are met. The importance of assumptions 2 and 3 becomes apparent after we introduce the basic circuit elements and the rules for analyzing interconnected elements.

However, we need to take a closer look at assumption 1. The question is, "How small does a physical system have to be to qualify as a lumped-parameter system?" We can get a quantitative handle on the question by noting that electric signals propagate by wave phenomena. If the wavelength of the signal is large compared to the physical dimensions of the system, we have a lumped-parameter system. The wavelength λ is the velocity divided by the repetition rate, or **frequency**, of the signal; that is, $\lambda = c/f$. The frequency f is measured in hertz (Hz). For example, power systems in the United States operate at 60 Hz. If we use the speed of light ($c = 3 \times 10^8$ m/s) as the velocity of propagation, the wavelength is 5×10^6 m. If the power system of interest is physically smaller than this wavelength, we can represent it as a lumped-parameter system and use circuit theory to analyze its behavior. How do we define *smaller*? A good rule is the *rule of 1/10th*: if the dimension of the system is 1/10th (or smaller) of the dimension of the wavelength, you have a lumped-parameter system. Thus, as long as the physical dimension of the power system is less than 5×10^5 m, we can treat it as a lumped-parameter system.

On the other hand, the propagation frequency of radio signals is on the order of 10^9 Hz. Thus the wavelength is 0.3 m. Using the rule of 1/10th, the relevant dimensions of a communication system that sends or receives radio signals must be less than 3 cm to qualify as a lumped-parameter system. Whenever any of the pertinent physical dimensions of a system under study approaches the wavelength of its signals, we must use electromagnetic field theory to analyze that system. Throughout this book we study circuits derived from lumped-parameter systems.

Problem Solving

As a practicing engineer, you will not be asked to solve problems that have already been solved. Whether you are trying to improve the performance of an existing system or creating a new system, you will be working on unsolved problems. As a student, however, you will devote much of your attention to the discussion of problems already solved. By reading about and discussing how these problems were solved in the past, and by solving related homework and exam problems on your own, you will begin to develop the skills to successfully attack the unsolved problems you'll face as a practicing engineer.

Some general problem-solving procedures are presented here. Many of them pertain to thinking about and organizing your solution strategy *before* proceeding with calculations.

1. *Identify what's given and what's to be found.* In problem solving, you need to know your destination before you can select a route for getting there. What is the problem asking you to solve or find? Sometimes the goal of the problem is obvious; other times you may need to paraphrase or make lists or tables of known and unknown information to see your objective.

   The problem statement may contain extraneous information that you need to weed out before proceeding. On the other hand, it may offer incomplete information or more complexities than can be handled given the solution methods at your disposal. In that case, you'll need to make assumptions to fill in the missing information or simplify the problem context. Be prepared to circle back and reconsider supposedly extraneous information and/or your assumptions if your calculations get bogged down or produce an answer that doesn't seem to make sense.
2. *Sketch a circuit diagram or other visual model.* Translating a verbal problem description into a visual model is often a useful step in the solution process. If a circuit diagram is already provided, you may need to add information to it, such as labels, values, or reference directions. You may also want to redraw the circuit in a simpler, but equivalent, form. Later in this text you will learn the methods for developing such simplified equivalent circuits.
3. *Think of several solution methods and decide on a way of choosing among them.* This course will help you build a collection of analytical tools, several of which may work on a given problem. But one method may produce fewer equations to be solved than another, or it may require only algebra instead of calculus to reach a solution. Such efficiencies, if you can anticipate them, can streamline your calculations considerably. Having an alternative method in mind also gives you a path to pursue if your first solution attempt bogs down.
4. *Calculate a solution.* Your planning up to this point should have helped you identify a good analytical method and the correct equations for the problem. Now comes the solution of those equations. Paper-and-pencil, calculator, and computer methods are all available for performing the actual calculations of circuit analysis. Efficiency and your instructor's preferences will dictate which tools you should use.
5. *Use your creativity.* If you suspect that your answer is off base or if the calculations seem to go on and on without moving you toward a solution, you should pause and consider alternatives. You may need to revisit your assumptions or select a different solution method. Or, you may need to take a less-conventional problem-solving approach, such as working backward from a solution. This text provides answers to all of the Assessment Problems and many of the Chapter Problems so that you may work backward when you get stuck. In the real world, you won't be given answers in advance, but you may have a desired problem outcome in mind from which you can work backward. Other creative approaches include allowing yourself to see parallels with

other types of problems you've successfully solved, following your intuition or hunches about how to proceed, and simply setting the problem aside temporarily and coming back to it later.

6. *Test your solution.* Ask yourself whether the solution you've obtained makes sense. Does the magnitude of the answer seem reasonable? Is the solution physically realizable? You may want to go further and rework the problem via an alternative method. Doing so will not only test the validity of your original answer, but will also help you develop your intuition about the most efficient solution methods for various kinds of problems. In the real world, safety-critical designs are always checked by several independent means. Getting into the habit of checking your answers will benefit you as a student and as a practicing engineer.

These problem-solving steps cannot be used as a recipe to solve every problem in this or any other course. You may need to skip, change the order of, or elaborate on certain steps to solve a particular problem. Use these steps as a guideline to develop a problem-solving style that works for you.

1.2 The International System of Units

Engineers compare theoretical results to experimental results and compare competing engineering designs using quantitative measures. Modern engineering is a multidisciplinary profession in which teams of engineers work together on projects, and they can communicate their results in a meaningful way only if they all use the same units of measure. The International System of Units (abbreviated SI) is used by all the major engineering societies and most engineers throughout the world; hence we use it in this book.

The SI units are based on six *defined* quantities:

- length
- mass
- time
- electric current
- thermodynamic temperature
- luminous intensity

TABLE 1.1 The International System of Units (SI)

Quantity	Basic Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	degree kelvin	K
Luminous intensity	candela	cd

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These quantities, along with the basic unit and symbol for each, are listed in Table 1.1. Although not strictly SI units, the familiar time units of minute (60 s), hour (3600 s), and so on are often used in engineering calculations. In addition, defined quantities are combined to form **derived** units. Some, such as force, energy, power, and electric charge, you already know through previous physics courses. Table 1.2 lists the derived units used in this book.

In many cases, the SI unit is either too small or too large to use conveniently. Standard prefixes corresponding to powers of 10, as listed in Table 1.3, are then applied to the basic unit. All of these prefixes are correct, but engineers often use only the ones for powers divisible by 3; thus centi, deci, deka, and hecto are used rarely. Also, engineers often select the prefix that places the base number in the range between 1 and 1000. Suppose that a time calculation yields a result of 10^{-5} s, that is, 0.00001 s. Most engineers would describe this quantity as 10 μs, that is, $10^{-5} = 10 \times 10^{-6}$ s, rather than as 0.01 ms or 10,000,000 ps.

✓ ASSESSMENT PROBLEMS

Objective 1—Understand and be able to use SI units and the standard prefixes for powers of 10

1.1 How many dollars per millisecond would the federal government have to collect to retire a deficit of \$100 billion in one year?

Answer: \$3.17/ms.

1.2 If a signal can travel in a cable at 80% of the speed of light, what length of cable, in inches, represents 1 ns?

Answer: 9.45".

NOTE: Also try Chapter Problems 1.1, 1.3, and 1.6.

TABLE 1.2 Derived Units in SI

Quantity	Unit Name (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electric resistance	ohm (Ω)	V/A
Electric conductance	siemens (S)	A/V
Electric capacitance	farad (F)	C/V
Magnetic flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

TABLE 1.3 Standardized Prefixes to Signify Powers of 10

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

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1.3 Circuit Analysis: An Overview

Before becoming involved in the details of circuit analysis, we need to take a broad look at engineering design, specifically the design of electric circuits. The purpose of this overview is to provide you with a perspective on where circuit analysis fits within the whole of circuit design. Even though this book focuses on circuit analysis, we try to provide opportunities for circuit design where appropriate.

All engineering designs begin with a need, as shown in Fig. 1.4. This need may come from the desire to improve on an existing design, or it may be something brand-new. A careful assessment of the need results in design specifications, which are measurable characteristics of a proposed design. Once a design is proposed, the design specifications allow us to assess whether or not the design actually meets the need.

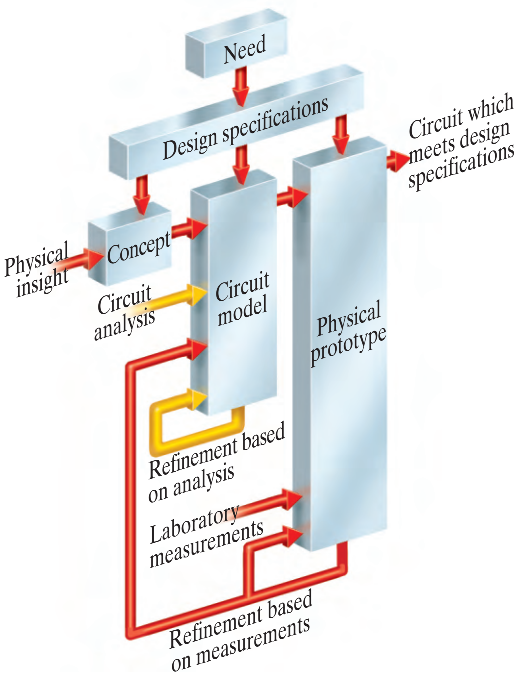


Figure 1.4 ▲ A conceptual model for electrical engineering design.

A concept for the design comes next. The concept derives from a complete understanding of the design specifications coupled with an insight into the need, which comes from education and experience. The concept may be realized as a sketch, as a written description, or in some other form. Often the next step is to translate the concept into a mathematical model. A commonly used mathematical model for electrical systems is a **circuit model**.

The elements that comprise the circuit model are called **ideal circuit components**. An ideal circuit component is a mathematical model of an actual electrical component, like a battery or a light bulb. It is important for the ideal circuit component used in a circuit model to represent the behavior of the actual electrical component to an acceptable degree of accuracy. The tools of **circuit analysis**, the focus of this book, are then applied to the circuit. Circuit analysis is based on mathematical techniques and is used to predict the behavior of the circuit model and its ideal circuit components. A comparison between the desired behavior, from the design specifications, and the predicted behavior, from circuit analysis, may lead to refinements in the circuit model and its ideal circuit elements. Once the desired and predicted behavior are in agreement, a physical prototype can be constructed.

The **physical prototype** is an actual electrical system, constructed from actual electrical components. Measurement techniques are used to determine the actual, quantitative behavior of the physical system. This actual behavior is compared with the desired behavior from the design specifications and the predicted behavior from circuit analysis. The comparisons may result in refinements to the physical prototype, the circuit model, or both. Eventually, this iterative process, in which models, components, and systems are continually refined, may produce a design that accurately matches the design specifications and thus meets the need.

From this description, it is clear that circuit analysis plays a very important role in the design process. Because circuit analysis is applied to circuit models, practicing engineers try to use mature circuit models so that the resulting designs will meet the design specifications in the first iteration. In this book, we use models that have been tested for between 20 and 100 years; you can assume that they are mature. The ability to model actual electrical systems with ideal circuit elements makes circuit theory extremely useful to engineers.

Saying that the interconnection of ideal circuit elements can be used to quantitatively predict the behavior of a system implies that we can describe the interconnection with mathematical equations. For the mathematical equations to be useful, we must write them in terms of measurable quantities. In the case of circuits, these quantities are voltage and current, which we discuss in Section 1.4. The study of circuit analysis involves understanding the behavior of each ideal circuit element in terms of its voltage and current and understanding the constraints imposed on the voltage and current as a result of interconnecting the ideal elements.

1.4 Voltage and Current

The concept of electric charge is the basis for describing all electrical phenomena. Let's review some important characteristics of electric charge.

- The charge is bipolar, meaning that electrical effects are described in terms of positive and negative charges.
- The electric charge exists in discrete quantities, which are integral multiples of the electronic charge, 1.6022×10^{-19} C.
- Electrical effects are attributed to both the separation of charge and charges in motion.

In circuit theory, the separation of charge creates an electric force (voltage), and the motion of charge creates an electric fluid (current).

The concepts of voltage and current are useful from an engineering point of view because they can be expressed quantitatively. Whenever positive and negative charges are separated, energy is expended. **Voltage** is the energy per unit charge created by the separation. We express this ratio in differential form as

$$v = \frac{dw}{dq}, \tag{1.1}$$

Definition of voltage

where

v = the voltage in volts,
w = the energy in joules,
q = the charge in coulombs.

The electrical effects caused by charges in motion depend on the rate of charge flow. The rate of charge flow is known as the **electric current**, which is expressed as

$$i = \frac{dq}{dt}, \tag{1.2}$$

Definition of current

where

i = the current in amperes,
q = the charge in coulombs,
t = the time in seconds.

Equations 1.1 and 1.2 are definitions for the magnitude of voltage and current, respectively. The bipolar nature of electric charge requires that we assign polarity references to these variables. We will do so in Section 1.5.

Although current is made up of discrete, moving electrons, we do not need to consider them individually because of the enormous number of them. Rather, we can think of electrons and their corresponding charge as one smoothly flowing entity. Thus, i is treated as a continuous variable.

One advantage of using circuit models is that we can model a component strictly in terms of the voltage and current at its terminals. Thus two physically different components could have the same relationship between the terminal voltage and terminal current. If they do, for purposes of circuit analysis, they are identical. Once we know how a component behaves at its terminals, we can analyze its behavior in a circuit. However, when developing circuit models, we are interested in a component's internal behavior. We might want to know, for example, whether charge conduction is taking place because of free electrons moving through the crystal lattice structure of a metal or whether it is because of electrons moving within the covalent bonds of a semiconductor material. However, these concerns are beyond the realm of circuit theory. In this book we use circuit models that have already been developed; we do not discuss how component models are developed.

1.5 The Ideal Basic Circuit Element

An **ideal basic circuit element** has three attributes: (1) it has only two terminals, which are points of connection to other circuit components; (2) it is described mathematically in terms of current and/or voltage; and (3) it cannot be subdivided into other elements. We use the word *ideal* to imply that a basic circuit element does not exist as a realizable physical component. However, as we discussed in Section 1.3, ideal elements can be connected in order to model actual devices and systems. We use the word *basic* to imply that the circuit element cannot be further reduced or subdivided into other elements. Thus the basic circuit elements form the building blocks for constructing circuit models, but they themselves cannot be modeled with any other type of element.

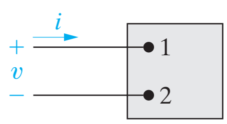


Figure 1.5 ▲ An ideal basic circuit element.

Figure 1.5 is a representation of an ideal basic circuit element. The box is blank because we are making no commitment at this time as to the type of circuit element it is. In Fig. 1.5, the voltage across the terminals of the box is denoted by v, and the current in the circuit element is denoted by i. The polarity reference for the voltage is indicated by the plus and minus signs, and the reference direction for the current is shown by the arrow placed alongside the current. The interpretation of these references given positive or negative numerical values of v and i is summarized in Table 1.4. Note that

TABLE 1.4 Interpretation of Reference Directions in Fig. 1.5

	Positive Value	Negative Value
v	voltage drop from terminal 1 to terminal 2	voltage rise from terminal 1 to terminal 2
	or	*or*
	voltage rise from terminal 2 to terminal 1	voltage drop from terminal 2 to terminal 1
i	positive charge flowing from terminal 1 to terminal 2	positive charge flowing from terminal 2 to terminal 1
	or	*or*
	negative charge flowing from terminal 2 to terminal 1	negative charge flowing from terminal 1 to terminal 2

algebraically the notion of positive charge flowing in one direction is equivalent to the notion of negative charge flowing in the opposite direction.

The assignments of the reference polarity for voltage and the reference direction for current are entirely arbitrary. However, once you have assigned the references, you must write all subsequent equations to agree with the chosen references. The most widely used sign convention applied to these references is called the **passive sign convention**, which we use throughout this book. The passive sign convention can be stated as follows:

> Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element (as in Fig.1.5), use a positive sign in any expression that relates the voltage to the current. Otherwise, use a negative sign.

◀ **Passive sign convention**

We apply this sign convention in all the analyses that follow. Our purpose for introducing it even before we have introduced the different types of basic circuit elements is to impress on you the fact that the selection of polarity references along with the adoption of the passive sign convention is *not* a function of the basic elements nor the type of interconnections made with the basic elements. We present the application and interpretation of the passive sign convention in power calculations in Section 1.6.

✓ ASSESSMENT PROBLEMS

Objective 2—Know and be able to use the definitions of *voltage* and *current*

1.3 The current at the terminals of the element in Fig. 1.5 is

$$i = 0, \qquad t < 0;$$

$$i = 20e^{-5000t}\ \text{A}, \quad t \geq 0.$$

Calculate the total charge (in microcoulombs) entering the element at its upper terminal.

Answer: 4000 μC.

NOTE: Also try Chapter Problem 1.9.

1.4 The expression for the charge entering the upper terminal of Fig. 1.5 is

$$q = \frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\ \text{C}.$$

Find the maximum value of the current entering the terminal if $\alpha = 0.03679\ \text{s}^{-1}$.

Answer: 10 A.

1.6 Power and Energy

Power and energy calculations also are important in circuit analysis. One reason is that although voltage and current are useful variables in the analysis and design of electrically based systems, the useful output of the system often is nonelectrical, and this output is conveniently expressed in terms of power or energy. Another reason is that all practical devices have limitations on the amount of power that they can handle. In the design process, therefore, voltage and current calculations by themselves are not sufficient.

We now relate power and energy to voltage and current and at the same time use the power calculation to illustrate the passive sign convention. Recall from basic physics that power is the time rate of expending or absorbing energy. (A water pump rated 75 kW can deliver more liters per second than one rated 7.5 kW.) Mathematically, energy per unit time is expressed in the form of a derivative, or

Definition of power ▶

$$\boxed{p = \frac{dw}{dt},} \tag{1.3}$$

where

$$\begin{aligned} p &= \text{the power in watts,} \\ w &= \text{the energy in joules,} \\ i &= \text{the time in seconds.} \end{aligned}$$

Thus 1 W is equivalent to 1 J/s.

The power associated with the flow of charge follows directly from the definition of voltage and current in Eqs. 1.1 and 1.2, or

$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right),$$

so

Power equation ▶

$$\boxed{p = vi} \tag{1.4}$$

where

$$\begin{aligned} p &= \text{the power in watts,} \\ v &= \text{the voltage in volts,} \\ i &= \text{the current in amperes.} \end{aligned}$$

Equation 1.4 shows that the **power** associated with a basic circuit element is simply the product of the current in the element and the voltage across the element. Therefore, power is a quantity associated with a pair of terminals, and we have to be able to tell from our calculation whether power is being delivered to the pair of terminals or extracted from it. This information comes from the correct application and interpretation of the passive sign convention.

If we use the passive sign convention, Eq. 1.4 is correct if the reference direction for the current is in the direction of the reference voltage drop across the terminals. Otherwise, Eq. 1.4 must be written with a minus sign. In other words, if the current reference is in the direction of a reference voltage rise across the terminals, the expression for the power is

$$p = -vi \tag{1.5}$$

The algebraic sign of power is based on charge movement through voltage drops and rises. As positive charges move through a drop in voltage, they lose energy, and as they move through a rise in voltage, they gain energy. Figure 1.6 summarizes the relationship between the polarity references for voltage and current and the expression for power.

We can now state the rule for interpreting the algebraic sign of power:

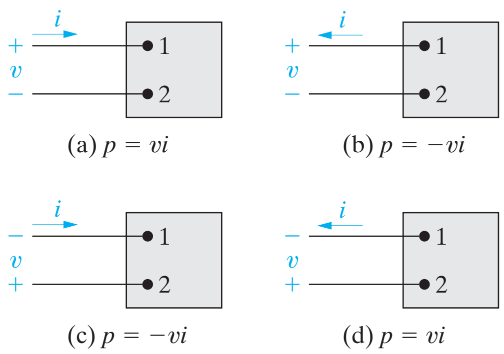


Figure 1.6 ▲ Polarity references and the expression for power.

◀ **Interpreting algebraic sign of power**

> If the power is positive (that is, if $p > 0$), power is being delivered to the circuit inside the box. If the power is negative (that is, if $p < 0$), power is being extracted from the circuit inside the box.

For example, suppose that we have selected the polarity references shown in Fig. 1.6(b). Assume further that our calculations for the current and voltage yield the following numerical results:

$$i = 4 \text{ A} \quad \text{and} \quad v = -10 \text{ V}.$$

Then the power associated with the terminal pair 1,2 is

$$p = -(-10)(4) = 40 \text{ W}.$$

Thus the circuit inside the box is absorbing 40 W.

To take this analysis one step further, assume that a colleague is solving the same problem but has chosen the reference polarities shown in Fig. 1.6(c). The resulting numerical values are

$$i = -4 \text{ A}, \qquad v = 10 \text{ V}, \qquad \text{and} \qquad p = 40 \text{ W}.$$

Note that interpreting these results in terms of this reference system gives the same conclusions that we previously obtained—namely, that the circuit inside the box is absorbing 40 W. In fact, any of the reference systems in Fig. 1.6 yields this same result.

✓ ASSESSMENT PROBLEMS

Objective 3—Know and use the definitions of *power* and *energy*; Objective 4—Be able to use the passive sign convention

1.5 Assume that a 20 V voltage drop occurs across an element from terminal 2 to terminal 1 and that a current of 4 A enters terminal 2.

a) Specify the values of v and i for the polarity references shown in Fig. 1.6(a)–(d).

b) State whether the circuit inside the box is absorbing or delivering power.

c) How much power is the circuit absorbing?

Answer: (a) Circuit 1.6(a): $v = -20$ V, $i = -4$ A;
circuit 1.6(b): $v = -20$ V, $i = 4$ A;
circuit 1.6(c): $v = 20$ V, $i = -4$ A;
circuit 1.6(d): $v = 20$ V, $i = 4$ A;

(b) absorbing;

(c) 80 W.

1.6 Assume that the voltage at the terminals of the element in Fig. 1.5 corresponding to the current in Assessment Problem 1.3 is

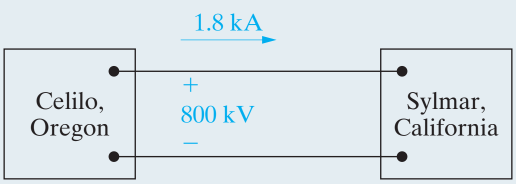
$$v = 0, \qquad t < 0;$$

$$v = 10e^{-5000t}\text{ kV}, \qquad t \geq 0.$$

Calculate the total energy (in joules) delivered to the circuit element.

Answer: 20 J.

1.7 A high-voltage direct-current (dc) transmission line between Celilo, Oregon and Sylmar, California is operating at 800 kV and carrying 1800 A, as shown. Calculate the power (in megawatts) at the Oregon end of the line and state the direction of power flow.



Answer: 1440 MW, Celilo to Sylmar.

NOTE: Also try Chapter Problems 1.12, 1.17, 1.24, and 1.26.

Summary

- The International System of Units (SI) enables engineers to communicate in a meaningful way about quantitative results. Table 1.1 summarizes the base SI units; Table 1.2 presents some useful derived SI units. (See pages 8 and 9.)
- Circuit analysis is based on the variables of voltage and current. (See page 11.)
- **Voltage** is the energy per unit charge created by charge separation and has the SI unit of volt ($v = dw/dq$). (See page 11.)
- **Current** is the rate of charge flow and has the SI unit of ampere ($i = dq/dt$). (See page 11.)
- The **ideal basic circuit element** is a two-terminal component that cannot be subdivided; it can be described mathematically in terms of its terminal voltage and current. (See page 12.)
- The **passive sign convention** uses a positive sign in the expression that relates the voltage and current at the terminals of an element when the reference direction for the current through the element is in the direction of the reference voltage drop across the element. (See page 13.)
- **Power** is energy per unit of time and is equal to the product of the terminal voltage and current; it has the SI unit of watt ($p = dw/dt = vi$). (See page 14.)
- The algebraic sign of power is interpreted as follows:
  - If $p > 0$, power is being delivered to the circuit or circuit component.
  - If $p < 0$, power is being extracted from the circuit or circuit component. (See page 15.)

Problems

Section 1.2

1.1 There are approximately 250 million passenger vehicles registered in the United States. Assume that the battery in the average vehicle stores 440 watt-hours (Wh) of energy. Estimate (in gigawatt-hours) the total energy stored in U.S. passenger vehicles.

1.2 The line described in Assessment Problem 1.7 is 845 mi in length. The line contains four conductors, each weighing 2526 lb per 1000 ft. How many kilograms of conductor are in the line?

1.3 The 4 giga-byte (GB $= 10^9$ bytes) flash memory chip for an MP3 player is 32 mm by 24 mm by 2.1 mm. This memory chip holds 1000 three-minute songs.

a) How many seconds of music fit into a cube whose sides are 1 mm?

b) How many bytes of memory are stored in a cube whose sides are 100 μm?

1.4 A hand-held video player displays 320×240 picture elements (pixels) in each frame of the video. Each pixel requires 2 bytes of memory. Videos are displayed at a rate of 30 frames per second. How many minutes of video will fit in a 30 gigabyte memory?

1.5 Some species of bamboo can grow 250 mm/day. Assume individual cells in the plant are 10 μm long.

a) How long, on average, does it take a bamboo stalk to grow 1 cell length?

b) How many cells are added in one week, on average?

1.6 One liter (L) of paint covers approximately 10 m^2 of wall. How thick is the layer before it dries? (*Hint*: $1 \text{ L} = 1 \times 10^6 \text{ mm}^3$.)

Section 1.4

1.7 A current of 1200 A exists in a copper wire, with a circular cross-section (radius = 1.5 mm). The current is due to free electrons moving through the wire at an average velocity of v meters/second. If the concentration of free electrons is 10^{29} electrons per cubic meter and if they are uniformly dispersed throughout the wire, then what is the average velocity of an electron?

1.8 In electronic circuits it is not unusual to encounter currents in the microampere range. Assume a 35 μA current, due to the flow of electrons. What is the average number of electrons per second that flow past a fixed reference cross section that is perpendicular to the direction of flow?

1.9 The current entering the upper terminal of Fig. 1.5 is

$$i = 24 \cos 4000t \text{ A}.$$

Assume the charge at the upper terminal is zero at the instant the current is passing through its maximum value. Find the expression for $q(t)$.

1.10 How much energy is extracted from an electron as it flows through a 6 V battery from the positive to the negative terminal? Express your answer in attojoules.

Sections 1.5–1.6

1.11 One 9 V battery supplies 100 mA to a camping flashlight. How much energy does the battery supply in 5 h?

1.12 Two electric circuits, represented by boxes A and B, are connected as shown in Fig. P1.12. The reference direction for the current i in the interconnection and the reference polarity for the voltage v across the inter connection are as shown in the figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or vice versa.

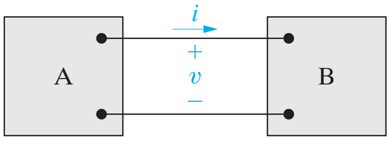
a) $i = 5$ A, $v = 120$ V

b) $i = -8$ A, $v = 250$ V

c) $i = 16$ A, $v = -150$ V

d) $i = -10$ A, $v = -480$ V

Figure P1.12



1.13 The references for the voltage and current at the terminal of a circuit element are as shown in Fig. 1.6(d). The numerical values for v and i are 40 V and −10 A

a) Calculate the power at the terminals and state whether the power is being absorbed or delivered by the element in the box.

b) Given that the current is due to electron flow, state whether the electrons are entering or leaving terminal 2.

c) Do the electrons gain or lose energy as they pass through the element in the box?

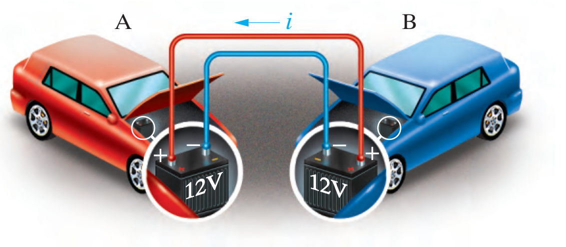
1.14 Repeat Problem 1.13 with a voltage of −60 V.

1.15 When a car has a dead battery, it can often be started by connecting the battery from another car across its terminals. The positive terminals are connected together as are the negative terminals. The connection is illustrated in Fig. P1.15. Assume the current i in Fig. P1.15 is measured and found to be 30 A.

a) Which car has the dead battery?

b) If this connection is maintained for 1 min, how much energy is transferred to the dead battery?

Figure P1.15



1.16 The manufacturer of a 9V dry-cell flashlight battery says that the battery will deliver 20 mA for 80 continuous hours. During that time the voltage will drop from 9 V to 6 V. Assume the drop in voltage is linear with time. How much energy does the battery deliver in this 80 h interval?

1.17 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = e^{-500t} - e^{-1500t} \text{ V},$$

$$i = 30 - 40e^{-500t} + 10e^{-1500t} \text{ mA}.$$

a) Find the power at $t = 1$ ms.

b) How much energy is delivered to the circuit element between 0 and 1 ms?

c) Find the total energy delivered to the element.

1.18 PSPICE The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = 400e^{-100t} \sin 200t \text{ V},$$

$$i = 5e^{-100t} \sin 200t \text{ A}.$$

a) Find the power absorbed by the element at $t = 10$ ms.

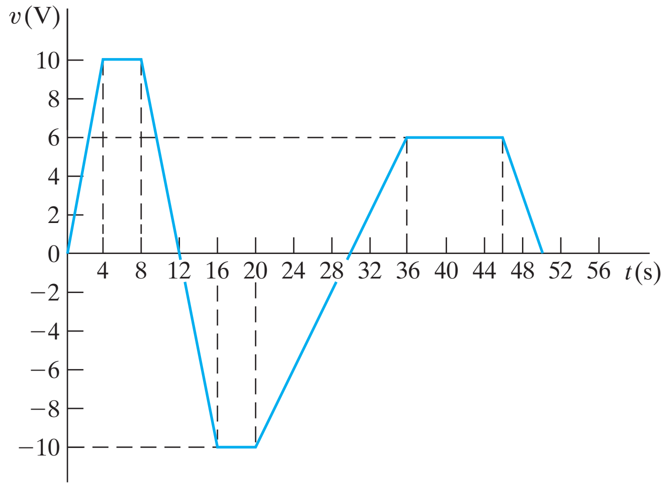
b) Find the total energy absorbed by the element.

1.19 The voltage and current at the terminals of the circuit element in Fig. 1.5 are shown in Fig. P1.19.

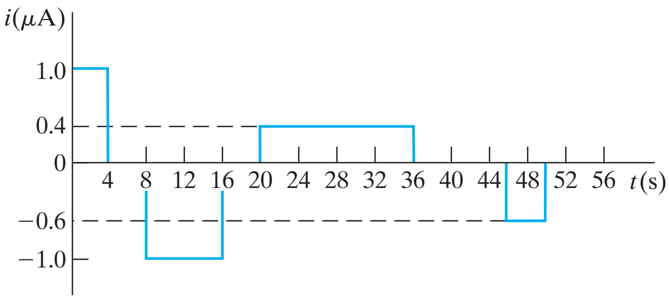
a) Sketch the power versus t plot for $0 \leq t \leq 50$ s.

b) Calculate the energy delivered to the circuit element at $t = 4$, 12, 36, and 50 s.

Figure P1.19



(a)



(b)

1.20 PSPICE The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = 75 - 75e^{-1000t} \text{ V},$$

$$i = 50\, e^{-1000t} \text{ mA}.$$

a) Find the maximum value of the power delivered to the circuit.

b) Find the total energy delivered to the element.

1.21 The voltage and current at the terminals of the element in Fig. 1.5 are
PSPICE

$$v = 36 \sin 200\pi t \text{ V}, \quad i = 25 \cos 200\pi t \text{ A}.$$

a) Find the maximum value of the power being delivered to the element.

b) Find the maximum value of the power being extracted from the element.

c) Find the average value of p in the interval $0 \leq t \leq 5$ ms.

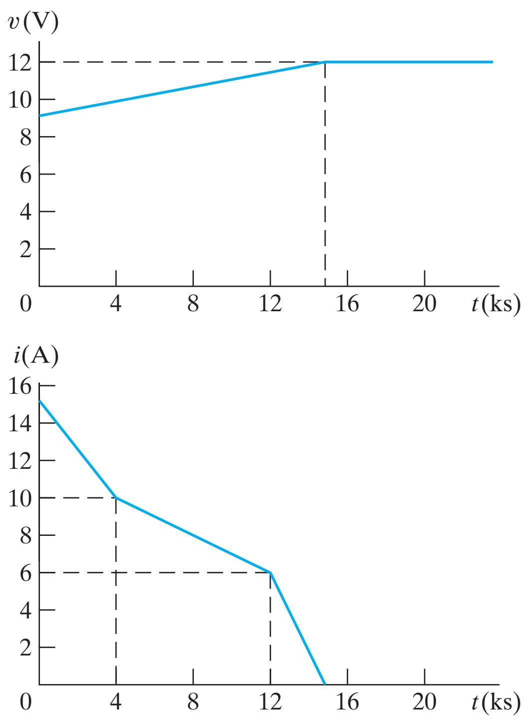
d) Find the average value of p in the interval $0 \leq t \leq 6.25$ ms.

1.22 The voltage and current at the terminals of an automobile battery during a charge cycle are shown in Fig. P1.22.
PSPICE

a) Calculate the total charge transferred to the battery.

b) Calculate the total energy transferred to the battery.

Figure P1.22



1.23 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are
PSPICE

$$v = (16{,}000t + 20)e^{-800t} \text{ V},$$

$$i = (128t + 0.16)e^{-800t} \text{ A}.$$

a) At what instant of time is maximum power delivered to the element?

b) Find the maximum power in watts.

c) Find the total energy delivered to the element in millijoules.

1.24 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$ and $t > 3$ s. In the interval between 0 and 3 s the expressions are
PSPICE

$$v = t(3 - t) \text{ V}, \quad 0 < t < 3 \text{ s};$$

$$i = 6 - 4t \text{ mA}, \quad 0 < t < 3 \text{ s}.$$

a) At what instant of time is the power being delivered to the circuit element maximum?

b) What is the power at the time found in part (a)?

c) At what instant of time is the power being extracted from the circuit element maximum?

d) What is the power at the time found in part (c)?

e) Calculate the net energy delivered to the circuit at 0, 1, 2 and 3 s.

1.25 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are
PSPICE

$$v = (10{,}000t + 5)e^{-400t} \text{ V}, \qquad t \geq 0;$$

$$i = (40t + 0.05)e^{-400t} \text{ A}, \qquad t \geq 0.$$

a) Find the time (in milliseconds) when the power delivered to the circuit element is maximum.

b) Find the maximum value of p in milliwatts.

c) Find the total energy delivered to the circuit element in millijoules.

1.26 The numerical values for the currents and voltages in the circuit in Fig. P1.26 are given in Table P1.26. Find the total power developed in the circuit.

Figure P1.26

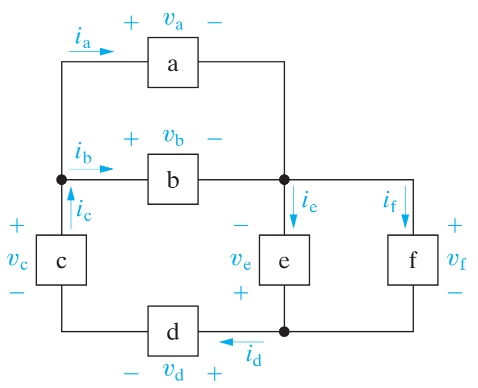


TABLE P1.26

Element	Voltage (mV)	Current (A)
a	150	0.6
b	150	−1.4
c	100	−0.8
d	250	−0.8
e	300	−2.0
f	−300	1.2

1.27 Assume you are an engineer in charge of a project and one of your subordinate engineers reports that the interconnection in Fig. P1.27 does not pass the power check. The data for the interconnection are given in Table P1.27.

a) Is the subordinate correct? Explain your answer.

b) If the subordinate is correct, can you find the error in the data?

Figure P1.27

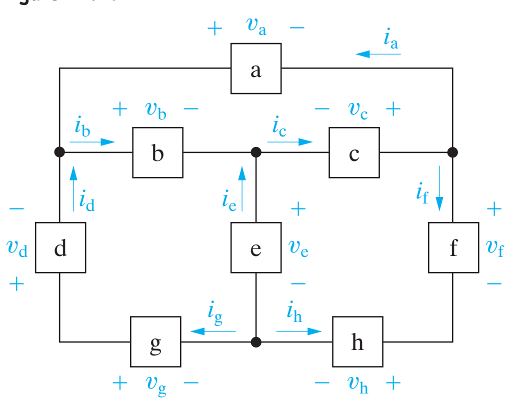


TABLE P1.27

Element	Voltage (kV)	Current (mA)
a	5.0	−150
b	2.0	250
c	3.0	200
d	−5.0	400
e	1.0	−50
f	4.0	350
g	−2.0	400
h	−6.0	−350

1.28 The numerical values of the voltages and currents in the interconnection seen in Fig. P1.28 are given in Table P1.28. Does the interconnection satisfy the power check?

Figure P1.28

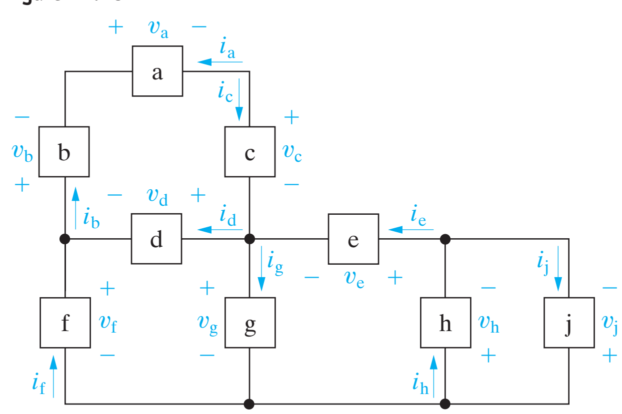


TABLE P1.28

Element	Voltage (V)	Current (μA)
a	36	250
b	44	−250
c	28	−250
d	−108	100
e	−32	150
f	60	−350
g	−48	−200
h	80	−150
j	80	−300

1.29 One method of checking calculations involving interconnected circuit elements is to see that the total power delivered equals the total power absorbed (conservation-of-energy principle). With this thought in mind, check the interconnection in Fig. P1.29 and state whether it satisfies this power check. The current and voltage values for each element are given in Table P1.29.

Figure P1.29

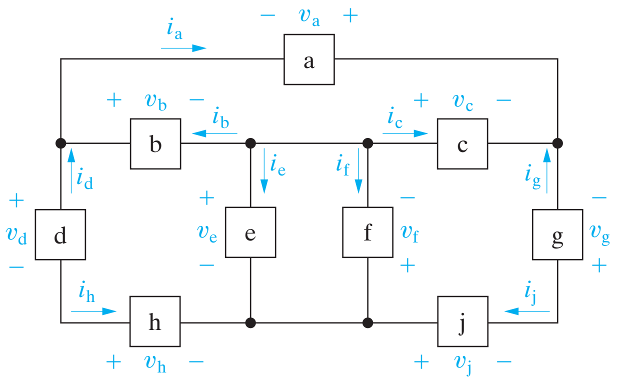


TABLE P1.29

Element	Voltage (V)	Current (mA)
a	1.6	80
b	2.6	60
c	−4.2	−50
d	1.2	20
e	1.8	30
f	−1.8	−40
g	−3.6	−30
h	3.2	−20
j	−2.4	30

1.30 a) In the circuit shown in Fig. P1.30, identify which elements are absorbing power and which are delivering power, using the passive sign convention.

b) The numerical values of the currents and voltages for each element are given in Table P1.30. How much total power is absorbed and how much is delivered in this circuit?

Figure P1.30

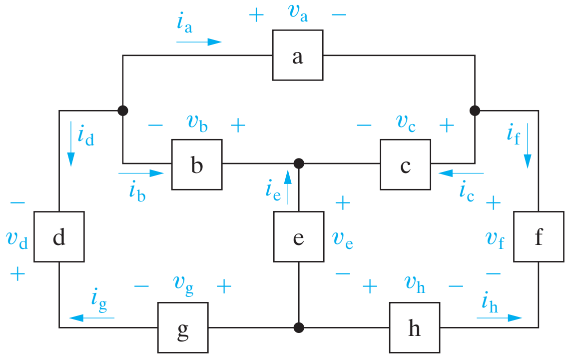


TABLE P1.30

Element	Voltage (mV)	Current (μA)
a	300	25
b	−100	10
c	−200	15
d	−200	−35
e	350	−25
f	200	10
g	−250	35
h	50	−10