Cross and Dot Products

The cross and dot products of vectors P = (1, 2, 3) and Q = (5, 0, 1) will be illustrated in four ways:
1. by hand
2. using the TI-85/86 calculator
3. using the HP-48G/GX calculator
4. using the TI-89 calculator

Determining cross products and dot products by hand:

P x Q = (1i + 2j + 3k) x (5i + 0j - 1k)
   = 5(i x i) + 0(i x j) - 1(i x k) +10(j x i) + 0(j x j) - 2(j x k) + 15(k x i) + 0(k x j) - 3(k x k)
   = 5(0) + 0(k) - 1(-j) + 10(-k) + 0(0) - 2(i) + 15(j) + 0(-i) - 3(0)
   = -2i + 16j - 10k
   = (-2,16,-10)

P • Q = (1i + 2j + 3k) • (5i + 0j - 1k) = (1)(5) + (2)(0) + (3)(-1) = 5 + 0 - 3 = 2

Determining cross products and dot products using the TI-85:

In the examples below, the calculator should be using rectangular coordinates for vectors (use MODE - RECTV).

There are two ways to accomplish these functions:
1. using the numerical values of the vectors to find the cross and dot products
2. saving the vectors as variables and using the variable names to find the cross and dot products

1.   **Using numerical values:**

Cross product:	Dot product:
2nd - VECTR - MATH - cross	*2nd - VECTR - MATH - dot*
cross([1, 2, 3] , [5, 0, -1]) *Enter*	*dot([1, 2, 3] , [5, 0, -1])* *Enter*
(-2.000, 16.000, -10.000) (the result)	*2.0000* (the result)

2.   **Using variables:**

Store vectors P and Q as follows:
[1, 2, 3] STO P   (or use any other name desired)
[5, 0, -1] STO Q

Now vector operations may be performed using P and Q, including cross and dot products as shown below:

Cross product:	Dot product:
2nd - VECTR - MATH - cross	*2nd - VECTR - MATH - dot*
cross(P, Q) Enter	*dot(P, Q) Enter*
(-2.000, 16.000, -10.000) (the result)	*2.0000* (the result)

Note: The TI-85/86 can also determine:
   A. the length of a vector: Use 2nd - VECTR - MATH - norm
   B. a unit vectors in the direction of a given vector: Use 2nd - VECTR - MATH - UNITV

Determining cross products and dot products using the TI-89:

In the examples below, the calculator should be using rectangular coordinates for vectors (use MODE - Vector Format - Rectangular).

There are two ways to accomplish these functions:
1. using the numerical values of the vectors to find the cross and dot products
2. saving the vectors as variables and using the variable names to find the cross and dot products

1.   **Using numerical values:**

Cross product:	Dot product:
Find the *crossP* function under CATALOG	Find the *dotP* function under CATALOG
crossP([1, 2, 3] , [5, 0, -1]) ***Enter***	*dotP([1, 2, 3] , [5, 0, -1])* ***Enter***
(-2.000, 16.000, -10.000) (the result)	*2.0000* (the result)

2.   **Using variables:**

Store vectors P and Q as follows:
[1, 2, 3] STO alpha p   (or use any other name desired)
[5, 0, -1] STO alpha q

Now vector operations may be performed using P and Q, including cross and dot products as shown below:

Cross product:	Dot product:
Find the *crossP* function under CATALOG	Find the *dotP* function under CATALOG
crossP(p, q) ***Enter***	*dot(P, Q)* ***Enter***
(-2.000, 16.000, -10.000) (the result)	*2.0000* (the result)

Note: The TI-89 can also determine:

A. the length of a vector: Use the function ***norm*** under CATALOG.

B. a unit vectors in the direction of a given vector: Use the function ***unitV*** under CATALOG

Determining cross products and dot products using the HP-48G/GX:

In the examples below, the calculator should be using rectangular coordinates for vectors (use MODES - Coordinate System - Rectangular).

1.   Using numerical values:

Cross product:	Dot product:
[1 2 3] Enter (load vector P onto the stack)	*[1 2 3] Enter*
[5 0 -1] Enter (load vector Q onto the stack)	*[5 0 -1] Enter*
MTH - VECTR - CROSS	*MTH - VECTR - DOT*
[-2.000 16.000 -10.000] (the result)	*2.0000* (the result)

2.   Using variables:

Cross product:	Dot product:
[1 2 3] α P STO (load vector P onto the stack)	*[1 2 3] α P STO* (load vector P onto the stack)
[5 0 -1] α Q STO (load vector Q onto the stack)	*[5 0 -1] α Q STO* (load vector Q onto the stack)
(Now vectors P and Q can be used as follows)	(Now vectors P and Q can be used as follows)
α P Enter (load vector P onto the stack)	*α P Enter* (load vector P onto the stack)
α Q Enter (load vector Q onto the stack)	*α Q Enter* (load vector Q onto the stack)
MTH - VECTR - CROSS	*MTH - VECTR - DOT*
[-2.000 16.000 -10.000] (the result)	*2.0000* (the result)

Also note that the length of a vector can be determined by loading it onto the stack and then using MTH - VECTR - ABS

Additional information on vector operations using the HP-48:

1) Entering vectors: Three ways to enter vectors are shown below:
   A) <u>Slow way</u>:
      [1 2 3] ENTER (loads vector P onto the stack by using square brackets)
   B) <u>Fast way</u>:
      1 SPC 2 SPC 3 →V3 (→V3 is under MTH - VECTR. Use →V2 for 2D vectors)
   C) <u>Alternate way</u>:
      1 ENTER 2 ENTER 3 ENTER →V3

   <u>Note</u>: You can also disassemble a vector and place its components on the stack with V →

2) <u>Vector Format</u>
   Vectors can be entered in rectangular coordinates (i, j, k) as shown above or you can use polar
   form (r, θ). To switch between the modes you have two good choices:
   A) Use the MTH - VECTR menu and go to the second page (with the NXT key). There you can
      select RECT, CYLIN, or SPHER. Changing the mode changes the display of all vectors on
      the stack.
   B) Use RIGHT SHIFT POLAR. This will toggle your display between rectangular and polar
      coordinates. When you are in CYLIN mode, you will see R/_Z at the top of the screen.

3) <u>Vector Operations</u> (the ABS, DOT, and CROSS functions are under the MTH - VECTR menu)

   ABS   Gives the magnitude of the vector currently on the stack, but you lose the vector.
            A quick way to look at the magnitude is just to switch to polar form (right shift
            POLAR). The first value will be the magnitude.

            <u>Note</u>: Here is an easy way to find a **<u>unit vector</u>**:
                     Enter the vector.
                     Press ENTER to make a duplicate of the vector on the stack.
                     Use ABS to replace the second copy of the vector with its length.
                     Press ÷ to divide the vector by its length (thus giving the unit vector).

   DOT   Gives the dot product of the two vectors currently on the stack. They must be the
            same size vectors (i.e., both 2D, both 3D, etc.).

            <u>Note</u>: Here is an easy way to find the angle between two vectors:
                     Enter the two vectors on the stack.
                     Find the unit vector from each vector (SWAP helps here).
                     Find the dot product of the two vectors (this yields cos(θ)).
                     Use ACOS to find the angle.

   CROSS   Gives the cross product of the two vectors currently on the stack. They must be the
            same size (i.e., 3D vectors).