EGR 140
Statics
File: Complex

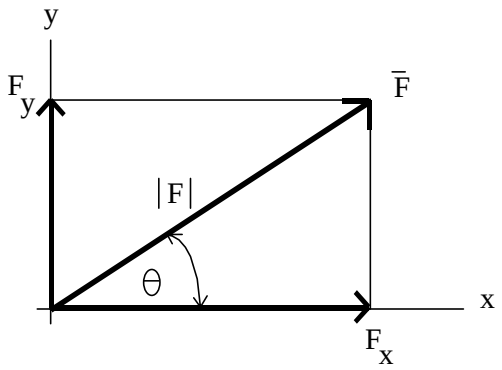
Complex Numbers

The representation of vectors using complex numbers, the conversion between rectangular and polar complex numbers, and calculations using complex numbers will be presented in four ways:

1. by hand
2. using the TI-85/86 calculator
3. using the HP-48G/GX
4. using the TI-89 calculator

Complex Numbers – Background and calculations by hand

Complex number can be used to represent two-dimensional vectors. The complex numbers may be stored in either polar or rectangular form.



Rectangular form: $\boxed{\bar{F} = F_x\mathbf{i} + F_y\mathbf{j}}$

where **i** is a unit vector along the x-axis and **j** is a unit vector along the y-axis.

Polar form: $\boxed{\bar{F} = |F|\angle\theta}$

where $|F|$ is the magnitude of vector $\bar{F}$ and

θ is the angle of vector $\bar{F}$ measured ***counterclockwise from the positive x - axis***

Converting between rectangular form and polar form:

Polar to Rectangular:

$F_x = |F|\cos(\theta)$

$F_y = |F|\sin(\theta)$

Rectangular to Polar:

$$|F| = \sqrt{F_x^2 + F_y^2} \qquad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Ex: Convert $\bar{F} = 100\angle 30$ N to rectangular form.

$F_x = 100\cos(30) = 86.6$

$F_y = 100\sin(30) = 50$

so $\bar{F} = 86.6\mathbf{i} + 50\mathbf{j}$ N

Ex: Convert $\bar{F} = 30\mathbf{i} + 40\mathbf{j}$ N to polar form.

$|F| = \sqrt{30^2 + 40^2} = 50$

$\theta = \tan^{-1}\left(\frac{40}{30}\right) = 53.13°$

so $\bar{F} = 50\angle 53.13$ N

Complex Numbers using the TI-85 or TI-86

Be sure that the calculator is in degree mode (press **2nd – MODE** to change the mode).

Complex numbers are stored as follows:
(100/_30) represents the polar number 100/30
(30,40) represents the rectangular number 30**i** + 40**j**

There are two ways to convert between complex number forms:
A) Using mode:

If the calculator is in rectangular mode then the following conversion can be made	
(100/_30) Enter	(the value entered in polar form)
(86.6,50)	(the result in rectangular form)

If the calculator is in polar mode then the following conversion can be made	
(30,40) Enter	(the value entered in rectangular form)
(50/_53.13)	(the result in polar form)

B) Using the →POL and →REC functions under the CPLX menu:
(This method works in any mode.)

(100/_30)	(the value entered in polar form)
2nd CPLX	
→REC Enter	
(86.6,50)	(the result in rectangular form)

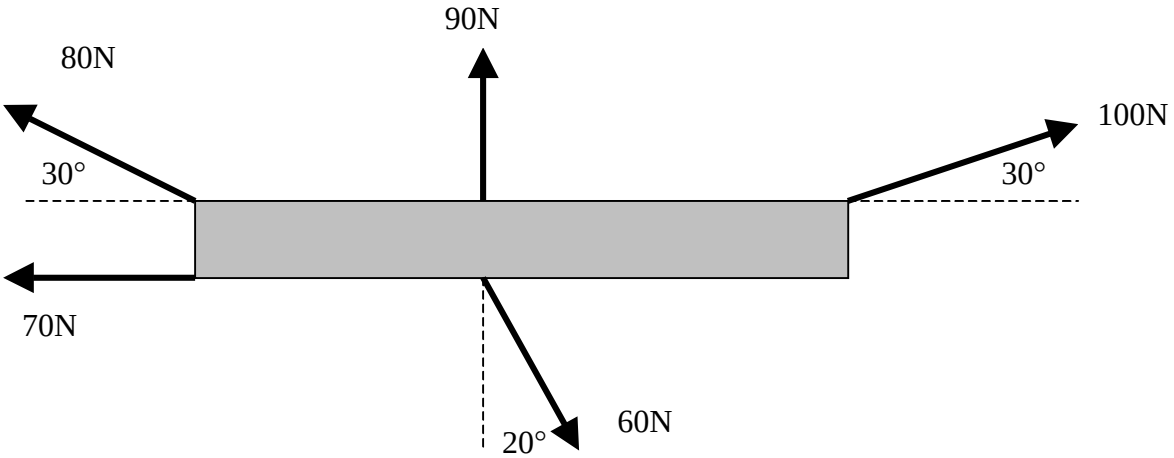
(30,40)	(the value entered in rectangular form)
2nd CPLX	
→POL Enter	
(50/_53.13)	(the result in polar form)

Using variables to store complex numbers with the TI-85/86:

Variables can be used to store complex numbers in either form. For example:

(100/_30) STO→ A	(store this polar number as variable A)
(30,40) STO→ B	(store this rectangular number as B)
A+B Enter	
(147.3/37.66)	(the result is in whatever form is specified by the mode)

<u>Performing calculations using complex numbers with the TI-85/86</u>:
Calculations can be performed using any combination of real and complex numbers (in any form). **<u>Example</u>**: Find the sum of the forces acting on the beam below:



Recalling that angles in polar numbers must be ***measured counterclockwise from the positive x-axis***, the sum of the forces above is easily found as follows:

(100/_30) + (0,90) + (80/_155) + (-70,0) + (60/_290) Enter ***(122.6/_106.8)*** (the result is in whatever form specified by the mode)

or

(100/_30) + (90/_90) + (80/_155) + (70/_180) + (60/_-70) Enter ***(122.6/_106.8)*** (the result is in whatever form specified by the mode)

Complex Numbers using the HP-48G or HP-48GX

Be sure that the calculator is in degree mode (use **MODES** to change the mode).

Complex numbers are stored as follows:
(100/_30) represents the polar number 100/30
(30,40) or (30 40) represents the rectangular number 30**i** + 40**j**

There are two ways to convert between complex number forms:
A) Using mode:

If the calculator is in rectangular mode then the following conversion can be made	
(100/_30) Enter	(the value entered in polar form)
(86.6 50)	(the result in rectangular form)

If the calculator is in polar mode then the following conversion can be made	
(30 40) Enter	(the value entered in rectangular form)
(50/_53.13)	(the result in polar form)

C) Using the RIGHT SHIFT POL to toggle between modes

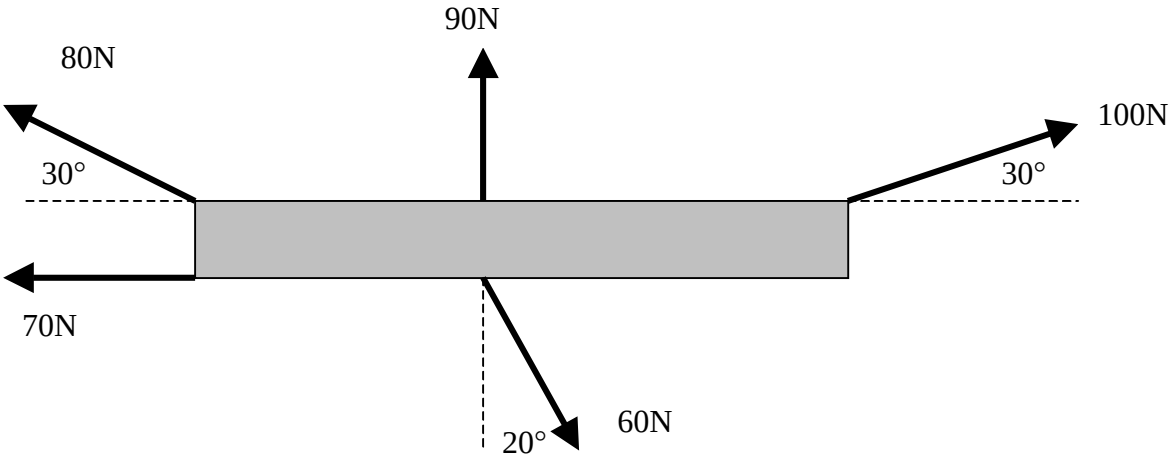
(100/_30)	(the value entered in polar form)
RIGHT SHIFT POLAR	
(86.6 50)	(the result has been toggled to rectangular form)
RIGHT SHIFT POLAR	
(100/_30)	(the result has been toggled back to polar form)

Using variables to store complex numbers with the HP-48G/GX:

Variables can be used to store complex numbers in either form. For example:

(100/_30) α A STO	(store this polar number as variable A)
(30 40) α B STO	(store this rectangular number as B)
(NOW A and B can be added as follows)	
α A Enter	
α B +	
(147.3/_37.66)	(the result is in whatever form is specified by the mode)

Performing calculations using complex numbers with the HP-48G/GX:
Calculations can be performed using any combination of real and complex numbers (in any form). **Example**: Find the sum of the forces acting on the beam below:



Recalling that angles in polar numbers must be ***measured counterclockwise from the positive x-axis***, the sum of the forces above is easily found as follows:

(100/_30) Enter	
(0 90) +	
(80/_155) +	
(-70 0) +	
(60/_290) +	
(122.6/_106.8)	(the result is in whatever form specified by the mode)

or

(100/_30) Enter	
(90/_90) +	
(80/_155) +	
(70/_180) +	
(60/_-70) +	
(122.6/_106.8)	(the result is in whatever form specified by the mode)

Complex Numbers using the TI-89

Be sure that the calculator is in degree mode (press **MODE** to change the mode).

Complex numbers are stored as follows:
(100/_30) represents the polar number 100/30
30 + 40i represents the rectangular number 30**i** + 40**j**

There are two ways to convert between complex number forms:
A) Using mode:

If the calculator is in rectangular mode then the following conversion can be made ***(100/_30) Enter*** (the value entered in polar form) ***86.6 + 50i*** (the result in rectangular form)

If the calculator is in polar mode then the following conversion can be made ***30 + 40i Enter*** (the value entered in rectangular form) ***(50/_53.13)*** (the result in polar form)

D) Using the →Polar and →Rect functions under CATALOG:
(This method works in any mode.)

(100/_30) (the value entered in polar form) ***CATALOG*** ***→Rect Enter*** ***86.6 + 50i*** (the result in rectangular form)

30 + 40i (the value entered in rectangular form) ***CATAlOG*** ***→Polar Enter*** ***(50/_53.13)*** (the result in polar form)

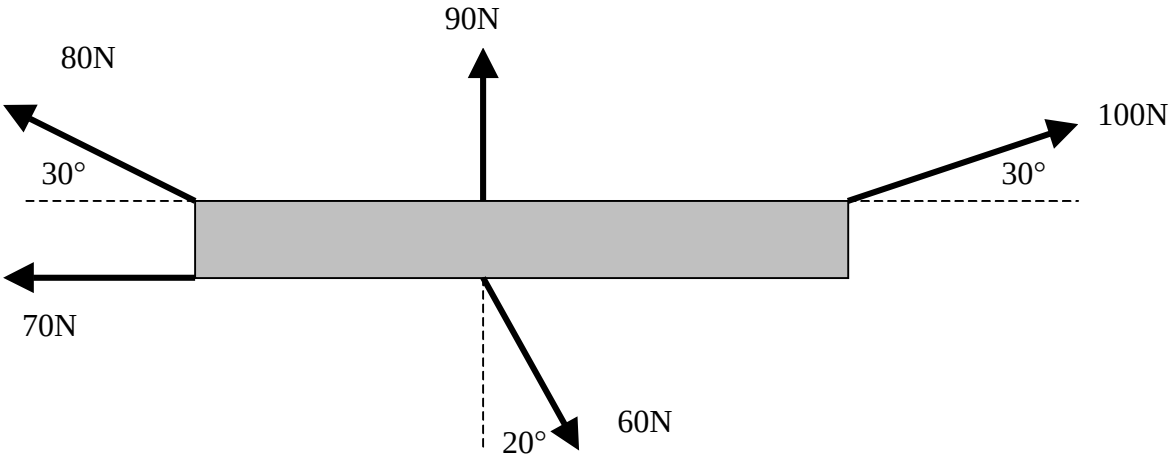
Using variables to store complex numbers with the TI-89:

Variables can be used to store complex numbers in either form. For example:

(100/_30) STO alpha a (store this polar number as variable a) ***30 + 40i STO alpha b*** (store this rectangular number as b) ***a+b Enter*** ***(147.3/37.66)*** (the result is in whatever form is specified by the mode)

Performing calculations using complex numbers with the TI-89:

Calculations can be performed using any combination of real and complex numbers (in any form). **Example**: Find the sum of the forces acting on the beam below:



Recalling that angles in polar numbers must be ***measured counterclockwise from the positive x-axis***, the sum of the forces above is easily found as follows:

(100/_30) + 90i + (80/_155) -70 + (60/_290) Enter   ***(122.6/_106.8)*** (the result is in whatever form specified by the mode)

or

(100/_30) + (90/_90) + (80/_155) + (70/_180) + (60/_-70) Enter   ***(122.6/_106.8)*** (the result is in whatever form specified by the mode)