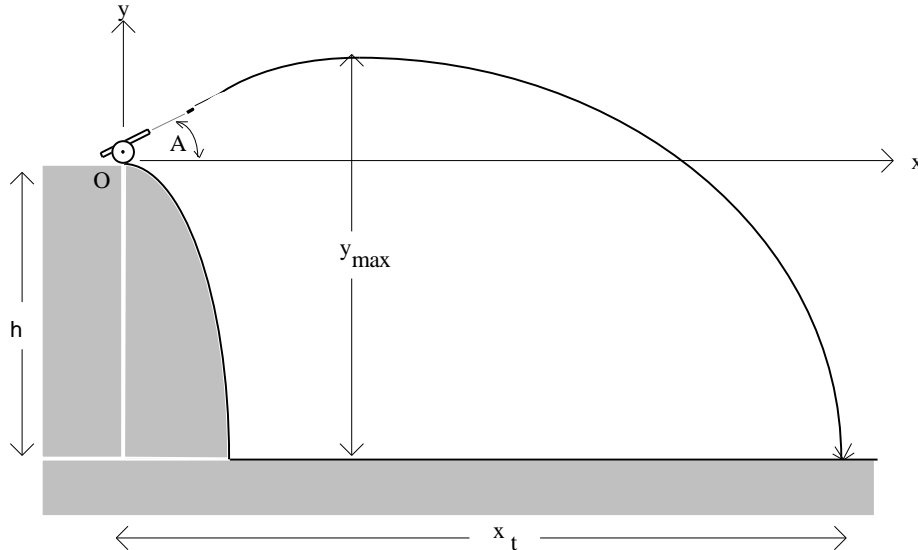


## Programming Assignment #3: Projectile Trajectory

A typical problem encountered in the study of dynamics is the trajectory problem. In the situation illustrated below, a projectile is fired from the edge of a cliff with an initial velocity,  $V_o$ , and a firing angle,  $A$ . The cliff has a height,  $h$ . It is desired to:

1. determine the distance,  $x_t$ , to the target
2. determine the highest elevation reached,  $y_{\max}$
3. determine the time to reach the target,  $t_t$
4. generate a table of  $(x,y)$  points to use in plotting the trajectory



### Background:

If  $V_o$  = initial velocity  
and  $A$  = firing angle (in degrees), then  
 $V_{xo}$  = initial horizontal velocity =  $V_o \cos(A)$   
 $V_{yo}$  = initial vertical velocity =  $V_o \sin(A)$

If  $g = -9.81 \text{ m/s}^2$  = acceleration due to gravity, then solving for  $t$  in the quadratic equation (the positive root)

$$-h = V_{yo}t + 0.5gt^2$$

will give the time to reach the target,  $t_t$  (i.e.,  $t = t_t$ ). Using this value of  $t_t$ , the distance to the target is

$$x_t = V_{xo}t_t$$

and the maximum height reached is

$$y_{\max} = h - \frac{(V_{yo})^2}{2g}$$

In order to generate a table of  $N(x, y)$  values, the final distance,  $x_t$ , can be divided into  $N-1$  even increments as shown below:

$$x = 0.0, \left(\frac{1}{N-1}\right)x_t, \left(\frac{2}{N-1}\right)x_t, \left(\frac{3}{N-1}\right)x_t, \dots, \left(\frac{N-2}{N-1}\right)x_t, x_t,$$

For example, if  $N = 21$ :  $x = 0.00, 0.05x_t, 0.10x_t, 0.15x_t, \dots, 0.95x_t, x_t$

Using the  $x$  values above, the corresponding values of  $t$  and  $y$  can be found using

$$t = \frac{x}{V_{x0}}$$

$$y = V_{y0}t + 0.5gt^2$$

### Program Requirements:

- The user of the program should be prompted to input four values from the keyboard. Also apply the restrictions indicated and allow the user to re-enter bad inputs):
  - Cliff height,  $h$ , in m  $h > 0$
  - Initial velocity,  $V_o$ , in m/s  $V_o > 0$
  - Angle,  $A$ , in degrees  $90^\circ > A > 0$
  - Number of point,  $N$ .  $100 \geq N > 5$
- Use functions for at least the following:
  - to convert an angle from degrees to radians
  - to find the two real roots of the quadratic equation when given the coefficients. The function might be called as: **RealRoots(A,B,C,Root1,Root2);**
- Use arrays to store the values of  $t$ ,  $x$ , and  $y$ . The arrays should be dimensioned for a max value of 100 points. The actual number of points,  $N$ , will be specified by the user.
- Give the user the option of:
  - Displaying a table of values on the computer screen
  - Sending the values to a data file so that they can be graphed using Excel

If option A is selected, the output to the screen should include a brief program description, the input values, and calculated values for  $x_t$ ,  $y_{max}$ , and  $t_t$ , and a table of  $(t,x,y)$  values. All outputs should be formatted and include units. The table might look as follows (lines are not required, but could be added for extra credit):

| time, t (s) | distance, x (m) | height, y (m) |
|-------------|-----------------|---------------|
| 0.000       | 0.0             | 0.0           |
| 1.134       | 283.6           | 91.7          |
| .           | .               | .             |
| .           | .               | .             |
| 22.673      | 5672.6          | -150.0        |

If option B is selected, only the values of  $t$ ,  $x$ , and  $y$  should be sent to the data file. Put commas between each value so that the file can be opened in Excel as a “commas delimited file”. Also prompt the user to enter the name of the output data file. The data file might look as follows:

```
0,0,0
1.134,283.6,91.7
.
.
.
22.673,5672.7,-150.0
```

5. Give the user the option of re-running the program.
6. Include a printout of the program and run the program for the following test cases:
  - A. Example 1 below with output to screen. Include a printout of the screen output.
  - B. Example 1 below with output to a data file. Open the data file with Excel and graph the trajectory (y versus x). Include a printout of the data file, and the Excel worksheet (both the table and the graph). The graph should be nicely formatted and labeled.
  - C. Repeat 6A using the inputs indicated corresponding to your last name in Table 1. Let N = the last two non-zero digits of your student ID. For example, if your student ID is 3247608, then N = 68 points.
  - D. Repeat 6B using the inputs indicated corresponding to your last name in Table 1. Let N = the last two non-zero digits of your student ID.

**Table 1: Input parameters:**

| Last Name begins with | $V_o$ (m/s) | A (degrees) | h (m) |
|-----------------------|-------------|-------------|-------|
| a-b                   | 180         | 20          | 110   |
| c-d                   | 180         | 30          | 115   |
| e-f                   | 180         | 40          | 120   |
| g-h                   | 220         | 20          | 125   |
| i-j                   | 220         | 30          | 130   |
| k-l                   | 220         | 40          | 135   |
| m-n                   | 260         | 20          | 140   |
| o-p                   | 260         | 30          | 145   |
| q-r                   | 260         | 40          | 150   |
| s-t                   | 300         | 20          | 155   |
| u-v                   | 300         | 30          | 160   |
| w-x                   | 300         | 40          | 165   |
| y-z                   | 340         | 20          | 170   |

**Example 1:**

You can test your program with the following data:

**Inputs:**

$$V_o = 180 \text{ m/s}$$

$$h = 150 \text{ m}$$

$$A = 30^\circ$$

$$N = 21 \text{ points}$$

**Outputs:**

$$t_t = 19.9 \text{ s}$$

$$x_t = 3100.0 \text{ m}$$

$$y_{\max} = 562.8 \text{ m}$$

(The values of t, x, y are not shown for this example).

**Extra Credit Suggestions:** (for a maximum of 10 additional points on the program grade)

1. Allow the user to select the units that he or she wishes to work with (e.g.,  $V_o$  could be in m/s, ft/s, or mph). The output should print the appropriate units.
2. Instead of using arrays, use the vector class in the Standard Template Library to store the values of  $t$ ,  $x$ , and  $y$ . Specify the vector size to be  $N$  after the user enters the value of  $N$ .
3. Give the user the option of entering inputs with the keyboard or reading them from a data file. If they are read from the data file, the program should still check to make sure that the inputs are valid.
4. Modify the program so that it will also allow the user to determine one or both of the following (for students who enjoyed Dynamics!):
  - a) Given  $V_o$ ,  $h$ , and the desired target distance,  $x_t$ , find  $A$ .
  - b) Given  $A$ ,  $h$ , and the desired target distance,  $x_t$ , find  $V_o$ .

Reference: Beer and Johnson, *Vector Mechanics for Engineers*
5. Use your imagination!