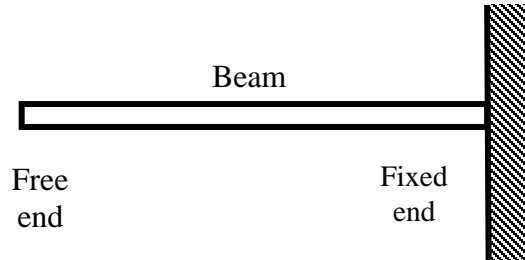


## Programming Assignment #2: Beam Deflection

The analysis of beams is an important topic in mechanical engineering. One type of beam considered is a **cantilever beam** in which one end of the beam is fixed and the other end of the beam is free.

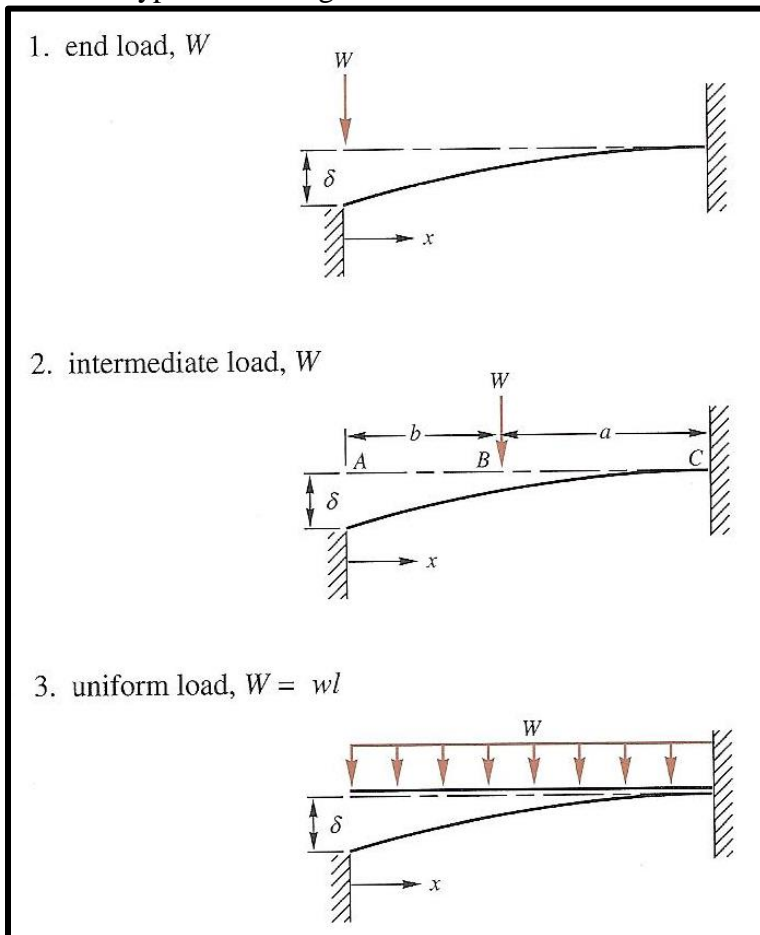


When a weight,  $W$ , (or load) is applied the free end of the beam, the beam deflects or bends. The amount of deflection,  $\delta$ , on the free end of the beam depends on the beams loading conditions.

There are many types of loading conditions, but three types are considered here:

- 1) **End load** - A single load,  $W$ , is applied at the free end of the beam
- 2) **Intermediate Load** - A single load,  $W$ , is applied at point B (somewhere between the free end and the fixed end)
- 3) **Uniform Load** - The load  $W$  is uniformly distributed along the length of the beam

The three types of loading are illustrated below:



$W$  = total weight of load (N)  
 $\delta$  = deflection caused by the load (m)  
 $x$  = distance from the free end of the beam (m)  
 $l = L$  = length of the beam (m)  
 $w$  = load density (N/m)  
For a uniform load,  $W = w \cdot L$  [note: (N/m)(m)=N]

The deflection for each beam can be calculated using the following relationships, assuming that the forces on the beam are coplanar, the beam is in static equilibrium, and that the weight of the beam may be neglected.

1) End Load: 
$$\delta = \left( \frac{-W}{6 \cdot E \cdot I} \right) (x^3 - 3 \cdot L^2 \cdot x + 2 \cdot L^3)$$

2) Intermediate Load: 
$$\delta = \begin{cases} \left( \frac{-W}{6 \cdot E \cdot I} \right) (-a^3 + 3 \cdot a^2 \cdot L - 3 \cdot a^2 \cdot x) & \text{for } 0 \leq x \leq b \\ \left( \frac{-W}{6 \cdot E \cdot I} \right) ((x - b)^3 - 3 \cdot a^2 \cdot (x - b) + 2 \cdot a^3) & \text{for } b < x \leq L \end{cases}$$

3) Uniform Load: 
$$\delta = \left( \frac{-W}{24 \cdot E \cdot I \cdot L} \right) (x^4 - 4 \cdot L^3 \cdot x + 3 \cdot L^4)$$

where:

E = Young's modulus of elasticity (N/m<sup>2</sup>)

I = moment of inertia (m<sup>4</sup>)

L = length of beam (m)

For the Intermediate Load, the following terms also apply:

a = distance from fixed end of beam to application of the load (m)

b = L - a

For the Uniform Load: Weight = (Weight Density)(Beam Length) or W = (w)(L)

Young's modulus of elasticity varies with different types of materials. The following are typical values:

Material	Young's modulus (N/m <sup>2</sup> )
Aluminum	6.90 x 10 <sup>10</sup>
Brass	11.7 x 10 <sup>10</sup>
Carbon Steel	20.5 x 10 <sup>10</sup>
Cast Iron	10.0 x 10 <sup>10</sup>
Oak	1.10 x 10 <sup>10</sup>
Stainless Steel	19.0 x 10 <sup>10</sup>
Titanium	11.4 x 10 <sup>10</sup>

It will be assumed that the beam under consideration is an I-beam with the following dimensions:

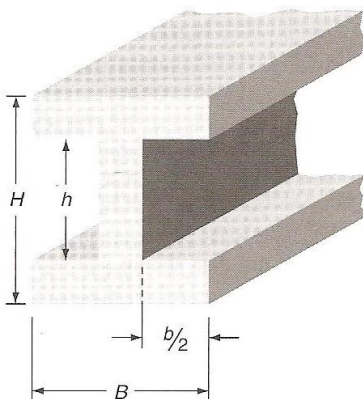


Figure 4.6 An I-beam

H = outer height of I-Beam (m)

h = inner height of I-Beam (m)

B = outer width of I-Beam (m)

b = width of both flanges of I-Beam (m)

The moment of inertia, I, for the I-Beam can be calculated as follows :

$$I = \frac{B \cdot H^3 - b \cdot h^3}{12} \text{ (m}^4\text{)}$$

**Program Requirements:**

Write a C++ program according to the following specifications

1. Display a brief description of the program.
2. Ask the user to specify the type of loading (end load, intermediate load, uniform load)
3. Ask the user to specify the type of material (give the user of menu of 7 material types listed above) and then use the given value of E.
4. Ask the user for the beam dimensions (L, B, b, H, and h). Use a function to find the moment of inertia.
5. If an end load or intermediate load is chosen, prompt the user to enter the weight of the load.
6. If a uniform load is selected, prompt the user to enter the weight density.
7. If an intermediate load is chosen, prompt the user to enter the value of b. (Note: This is the b on the intermediate load diagram where  $a + b = L$ , not the b used in the beam dimension. You might want to rename it.)
8. Create 3 functions to calculate  $\delta$  for the three types of loading.
9. Prompt the user to specify the number of points, N (in addition to  $x = 0$ ) to use in a table of x and  $\delta$  values.
10. Neatly display the following:
  - A brief description of the program.
  - The type of loading and type of material.
  - All input quantities (both variable and name) with units. (Note: Re-display these values. Showing the input value next to the input prompt is not sufficient.)
  - Also display I and E (with units).
  - Display a table of values of x and  $\delta$  with an appropriate table heading (with units) and with all values nicely aligned.

Other important notes:

- User prompts should include both symbol and name of variable and unit.
- Use 3 digits after the decimal point for most results. For results less than 0.01, use scientific notation. Otherwise used fixed notation.
- Check all inputs for errors. If any inputs are entered incorrectly, allow the user to correct them. Error checks include:
  - Incorrect menu choices
  - All beam dimensions, weights, and weight density must be positive
  - $H > h$ ,  $B > b$ ,  $L > a$ , and  $a > 0$ .
- Give the user the option of re-running the program

**Example:** End Load, Carbon Steel I-Beam with  $B = 0.02$  m,  $H = 0.04$  m,  $b = 0.015$  m,  $h = 0.03$  m,  $W = 125$  N,  $L = 2$  m, and  $N = 10$ . (Note: required formatting not used)

Distance from end of beam, x (m)	deflection, d (m)
0.000E+00	-2.230E-02
2.000E-01	-1.897E-02
4.000E-01	-1.570E-02
6.000E-01	-1.257E-02
8.000E-01	-9.633E-03
1.000E+00	-6.969E-03
1.200E+00	-4.638E-03
1.400E+00	-2.709E-03
1.600E+00	-1.249E-03
1.800E+00	-3.233E-04
2.000E+00	0.000E+00

**Test Cases**

Run the program for the following test cases and turn in printouts of the results with your report.

Loading	Material	L (m)	H (m)	h (m)	B (m)	b (m)	N	W (N)	W (N/m)	a (m)
End	Carbon Steel	2	0.04	0.03	0.02	0.015	10	125		
Intermediate	Aluminum	10	0.05	0.04	0.025	0.02	20	500		4
Uniform	Brass	0.5	0.06	0.04	0.03	0.024	15		300	

**Extra Credit Suggestions:** (up to 10 additional points)

1. Add lines to table.
2. Display sketches of the beam configuration and I-Beam shape with variables labeled to be sure that the user understands what you are asking for.
3. Give the user the option of using different units and/or prefixes.
4. Include additional functions.
5. Have the user enter the material type as a string (any combination of upper and lower case letters should be accepted from the user).
6. Have the user enter the material type as a string and then read the corresponding value of E from a data file.
7. Use your imagination!

**Reference:** Programming for Engineers & Scientists, 3<sup>rd</sup> Edition, by Nyhoff and Leestma, pp 232 – 239 (ISBN 0-02-388655-2)