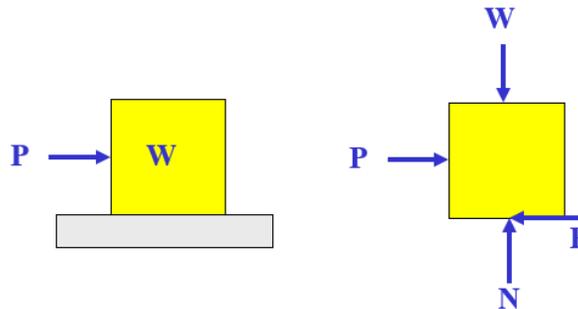


Programming Assignment #1: Static Friction

Static friction is typically introduced in Physics and Statics courses. A common problem to consider is that of a block resting on a plane.

Background:

Consider the diagram shown below on the left. The block rests on a surface where friction will be present between the block and the surface. A force P is applied to attempt to move the block. The FBD on the right shows the familiar reaction, including a normal force, N , and a frictional force, F . *Note that F always opposes the direction of motion.*



If P increases until the block is about to move (impending motion), then $F = F_m$ (maximum friction force). The maximum friction force, F_m , is controlled by the normal force, N , and the roughness of the material.

$$F_m = \mu_s N \text{ where } \mu_s = \textit{the coefficient of static friction}$$

Some typical values for μ_s are listed below in Table 1 (although they can vary considerably):

Table 1: Typical values for μ_s

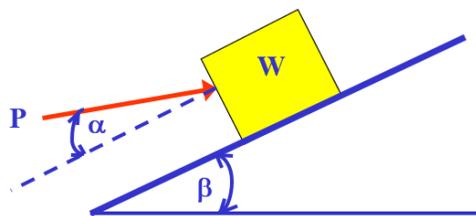
Surfaces	μ_s
metal on metal	0.20
metal on wood	0.25
metal on stone	0.28
metal on leather	0.30
wood on wood	0.23
wood on leather	0.32
stone on stone	0.50
rubber on concrete	0.75
earth on earth	0.80

Some notes regarding μ_s :

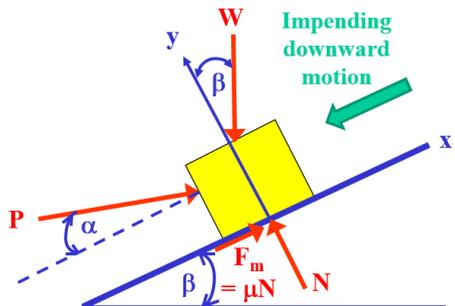
- μ_s is independent of surface area (a good approximation)
- μ_s is unitless and can be used to calculate friction forces with either SI or US units
- $0 \leq \mu_s \leq 1$

Impending upward motion versus impending downward motion

In the diagram shown below, there is a range of values of P for which the block will be in equilibrium (not moving uphill or downhill).



- The **minimum value of P (P_{down})** is when the block is about to begin sliding down the incline (*impending downward motion*). In this case the friction force is directed up the incline (opposing motion).



Equilibrium equations for impending downward motion :

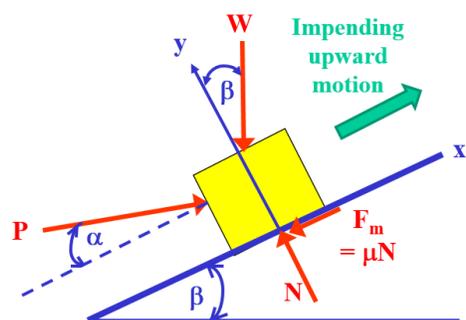
$$\sum F_x = 0 = P \cos(\alpha) + \mu N - W \sin(\beta)$$

$$\sum F_y = 0 = -P \sin(\alpha) + N - W \cos(\beta)$$

Solving for P yields :

$$P = \frac{W[\sin(\beta) - \mu \cos(\beta)]}{\cos(\alpha) + \mu \sin(\alpha)} \quad (P_{\text{down}})$$

- The **maximum value of P (P_{up})** is when the block is about to begin moving up the incline (*impending upward motion*). In this case the friction force is directed down the incline (opposing motion).



Equilibrium equations for impending downward motion :

$$\sum F_x = 0 = P \cos(\alpha) - \mu N - W \sin(\beta)$$

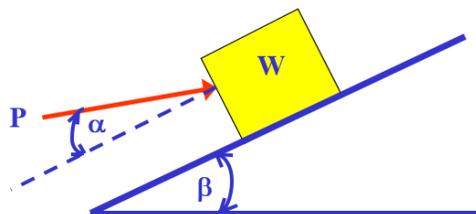
$$\sum F_y = 0 = -P \sin(\alpha) + N - W \cos(\beta)$$

Solving for P yields :

$$P = \frac{W[\sin(\beta) + \mu \cos(\beta)]}{\cos(\alpha) - \mu \sin(\alpha)} \quad (P_{\text{up}})$$

Program Requirements:

Write a C++ program to analyze the block and plane illustrated below according to the following specifications:



1. User inputs: The user should be prompted (with clear instructions) to input the values for P, W (or m), α , and β . Specifically:
 - The user should be asked whether they want to work in US units or SI units.
 - The force applied, P, should be entered in either:
 - N or kN (for SI units)
 - lb or kips (for US units) Note: 1 kip = 1 “kilopound” = 1000 lb
 - The angle of the force, α , and the angle of the incline, β , should be entered in either degrees or radians.
 - The user may enter either the weight or the mass of the block.
 - If SI units were chosen, enter mass in kg or weight in N or kN. Mass should be converted to weight. Recall that $W = mg$, where $g = 9.81 \text{ m/s}^2$.
 - If US units were chosen, enter mass in slugs or weight in lb or kips. Mass should be converted to weight. Recall that $W = mg$, where $g = 32.2 \text{ ft/s}^2$.
2. Finding μ_s : Determine the coefficient of static friction by prompting the user to enter the material for the block and the material for the plane (or choose the material from a menu). Valid block and plane materials include certain combinations of metal, wood, stone, leather, earth, rubber, and concrete. Using Table 1, determine the correct value of μ_s . Notes:
 - In Table 1, the materials may be listed in either order. For example, $\mu_s = 0.25$ for **wood on metal** (listed) and $\mu_s = 0.25$ for **metal on wood** (not specifically listed).
 - Only the combinations of valid materials listed are valid. For example, “leather on leather” is not listed in the table, so it is not valid.
3. Testing for valid inputs: If any invalid inputs occur, display a descriptive error message
 - Test for $-30^\circ < \alpha < 30^\circ$
 - Test for $0^\circ < \beta < 75^\circ$
 - If any units are entered as strings or characters, they must be properly spelled.
 - Only the allowable materials may be used and they must be properly spelled.
 - Values for P, W, and m must be positive.
4. Perform the following calculations:
 - Determine the minimum value of P_{up} (in lb or N) for impending **upward** motion.
 - Determine the minimum value of P_{down} (in lb or N) for impending **downward** motion.
 - If this value is positive, it means that the block will slide down on its own with $P_{\text{down}} = 0$.
 - If this value is negative, it means that the block will not slide down on its own with $P_{\text{down}} = 0$.
5. Displaying results: ***Redisplay all inputs*** and display all outputs in a neat format with clear names, symbols, and units. Specifically:
 - Display the following: P, W, m, α , β , block material, plane material, μ_s , minimum P for impending upward motion, and the minimum P for impending downward motion
 - Use 2 digits after the decimal point for all forces or weights and 1 digit after the decimal point for all angles.
 - For the value of P entered, state whether the block will:
 - Move upward (if $P \geq P_{\text{up}}$)
 - Move downward (if $P < P_{\text{down}}$)
 - Not move ($P_{\text{down}} < P < P_{\text{up}}$)
 - If $P = 0$, state whether the block will slide downward on its own or not (it will slide down on its own if $P_{\text{down}} > 0$)

Example:

Inputs: $\alpha = 18^\circ$, $\beta = 35^\circ$, P = 100 lb, W = 75 lb, block material = wood, plane material = metal:

Results: $\mu_s = 0.25$, P_{down} for downward motion = 26.90 lb, P_{up} for upward motion = 66.81 lb, the block will move upward for P = 100 lb. The block will slide down on its own if P = 0.

Program Testing: Print your results and include them with your report for the following 6 test cases:

Test #	System of Units	α	β	W or m	P	Block Material	Plane Material
1	US	18°	35°	75 lb	100 lb	wood	metal
2	US	0.3 rad	0.6 rad	0.3 slugs	50 lb	rubber	concrete
3	US	-10°	35°	0.7 kips	200 lb	metal	leather
4	SI	18°	35°	300 N	200 N	stone	metal
5	SI	0.3 rad	0.6 rad	0.6 kN	1.2 kN	earth	earth
6	SI	-10°	35°	75 kg	200 N	wood	wood

Note that Test#1 is the same as the example on the previous page, so you can check your results. It is also recommended that you test examples of various types of illegal inputs, but you do not need to include the results of these tests in your report.

Extra Credit Suggestions: (up to 10 additional points)

1. Instead of using a menu, use strings and require the user to enter the plane and block material. The user should be able to enter the material in all lowercase letters, all uppercase letters, or with the first letter only in uppercase. For example, wood, WOOD, and Wood are valid entries, but wood is invalid.
2. Rather than terminate the program for invalid inputs, display an error message for invalid inputs and allow the user to correct the inputs. This can be done fairly easily using do while loops (covered in Ch. 5).
3. Use a loop to allow the user to re-run the program.
4. Display an image of a block on an inclined plane at the start of the program.
5. Allow the user to use additional units, such as mass in g and force in oz.
6. Use your imagination!