Rocket City Math League

Senior Division

2010-2011
Inter-School Test

Answers must be written inside the answer boxes on the separate answer sheet. All answers must be written in exact, reduced, simplified, and rationalized form. **No calculators, books, or other aides may be used. You will be allowed 30 minutes to complete the test. Each team may only submit <u>one</u> official answer sheet.**

1. How many different ways can the letters in the word GRISSOM be arranged if the first letter has to be a G? *(1 point)*

2. Larry the Lobster wants to see how many pushups he can do. He asks Sandy to record the number of pushups he does. Sandy decides to complicate things and instead writes that the number of pushups Larry did is equal to $x^4 + 2x^3 + 3x^2 + 5x + 7$, where x=7. If Sandy is telling the truth, how many pushups did Larry do? *(1 point)*

3. If $f(x) = x^2 + 4x$ and $g(x) = 3x - 5$, find $f(g(x))$. *(1 point)*

4. Find all real solutions to the equation: $\ln x + \ln(x+2) = \ln(x+6)$ *(1 point)*

5. Find the sum of the cubes of the first 10 prime numbers. *(1 point)*

6. Steven is trying to enter his safe to get some space candy, but he forgot his code. He remembers that the code is the smallest 5-digit number that has all prime digits and is divisible by 13. What is the value of the code that will allow Steven to get his space candy? *(2 points)*

7. Find the sum of the units digits of each term of the sequence: $1^{2010}, 2^{2010}, 3^{2010}, \ldots, 9^{2010}$. *(2 points)*

8. Point C is equidistant from points A (2, 4, -3) and B (-3, 5, 1). If the y coordinate of C is 2 times the x coordinate and the z coordinate of C is 1 more than the x coordinate, give the coordinates of Point C. *(2 points)*

9. Solid A is a sphere; Solid B is a regular octahedron; Solid C is a right circular cylinder with height twice as large as the base radius. The greatest distance from the center of each solid to a point on the surface of that solid is 3. Find the value of $\frac{A}{C} \times B$ if each letter represents the volume of the solid with the corresponding letter. *(2 points)*

10. Evaluate: $\lim_{x\to\infty}\left(3^x + \left(3^x\right)^2\right)^{\frac{1}{x}}$ *(3 points)*

11. What is the smallest positive value of x for which $\sin\left(\frac{x}{10}\right) = \sin\left(\frac{x}{5}\right)$? *(3 points)*

12. Find the value of $R \cdot O \cdot C \cdot K \cdot E \cdot T \cdot C \cdot I \cdot T \cdot Y$ if R = the length of the altitude of an equilateral triangle with side length 4; O= the probability of rolling a number less than 2 on a 6-sided dice; C= the length of the radius of a circle with area 72π; K= the area of the regular hexagon with apothem $4\sqrt{3}$; E= the probability that if Zach rolls 2 6-sided dice, the sum of the 2 numbers will be 10; T= the sum of the solutions of the equation $x^2 - 7x = 5$; $I = \sqrt{497}$ rounded to the nearest tenth; and Y= the least common multiple of 64 and 55. *(3 points)*

13. Given that R, C, M, and L are positive integers, consider the set of possible solutions to the inequality : $14 \le R + C + M + L \le 18$. If one of these solutions is chosen at random, what is the probability that R + C + M + L is a multiple of 5? *(4 points)*

14. An ellipse with equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ has foci at (0, 1) and (6, 1) and passes through the point (3, -3). Find the value of a + b + h + k. *(4 points)*

15. Find $\sum_{x=4}^{\infty} \frac{4x^2 - 20x + 4}{x^4 + 2x^3 - 7x^2 - 8x + 12}$. *(5 points)*